2. Landau theory

For a particle of mass m_x traversing a thickness of material δx, the Landau probability distribution may be written in terms of the universal Landau function $\phi(\lambda)$ as[1]:

$$f(\epsilon, \delta x) = \frac{1}{\xi}\phi(\lambda)$$

where

$$\begin{aligned}
\phi(\lambda) &= \frac{1}{2\pi i}\int_{c+i\infty}^{c-i\infty} \exp\left(u \ln u + \lambda u\right) du \qquad c \geq 0 \\
\lambda &= \frac{\epsilon - \bar{\epsilon}}{\xi} - \gamma' - \beta^2 - \ln\frac{\xi}{E_{\max}} \\
\gamma' &= 0.422784\ldots = 1 - \gamma \\
\gamma &= 0.577215\ldots \text{(Euler's constant)} \\
\bar{\epsilon} &= \text{average energy loss} \\
\epsilon &= \text{actual energy loss}
\end{aligned}$$

2.1. Restrictions

The Landau formalism makes two restrictive assumptions:

1. The typical energy loss is small compared to the maximum energy loss in a single collision. This restriction is removed in the Vavilov theory (see section 3).

2. The typical energy loss in the absorber should be large compared to the binding energy of the most tightly bound electron. For gaseous detectors, typical energy losses are a few keV which is comparable to the binding energies of the inner electrons. In such cases a more sophisticated approach which accounts for atomic energy levels[4] is necessary to accurately simulate data distributions. In GEANT, a parameterised model by L. Urbán is used (see section 5).

In addition, the average value of the Landau distribution is infinite. Summing the Landau fluctuation obtained to the average energy from the dE/dx tables, we obtain a value which is larger than the one coming

from the table. The probability to sample a large value is small, so it takes a large number of steps (extractions) for the average fluctuation to be significantly larger than zero. This introduces a dependence of the energy loss on the step size which can affect calculations.

A solution to this has been to introduce a limit on the value of the variable sampled by the Landau distribution in order to keep the average fluctuation to 0. The value obtained from the GLANDO routine is:

$$\delta dE/dx = \epsilon - \bar{\epsilon} = \xi(\lambda - \gamma' + \beta^2 + \ln\frac{\xi}{E_{\max}})$$

In order for this to have average 0, we must impose that:

$$\bar{\lambda} = -\gamma' - \beta^2 - \ln\frac{\xi}{E_{\max}}$$

This is realised introducing a $\lambda_{\max}(\bar{\lambda})$ such that if only values of $\lambda \leq \lambda_{\max}$ are accepted, the average value of the distribution is $\bar{\lambda}$.