Package ‘skewMLRM’

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Type Package

Title Estimation for Scale-Shape Mixtures of Skew-Normal Distributions

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Description Provide data generation and estimation tools for the multivariate scale mixtures of normal presented in Lange and Sinsheimer (1993) <doi:10.2307/1390698>, the multivariate scale mixtures of skew-normal presented in Zeller, Lachos and Vilca (2011) <doi:10.1080/02664760903406504>, the multivariate skew scale mixtures of normal presented in Louredo, Zeller and Ferreira (2021) <doi:10.1007/s13571-021-00257-y> and the multivariate scale mixtures of skew-normal-Cauchy presented in Kahrari et al. (2020) <doi:10.1080/03610918.2020.1804582>.

Depends R (>= 4.0.0), stats, foreach

Imports moments, clusterGeneration, doParallel, parallel, MASS, mvtnorm, matrixcalc

Suggests sn

License GPL (>= 2)

NeedsCompilation no

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Contents

`choose2` *Select a distribution in the MSMN, MSSMN, MSMSN or/and MSMSNC classes and perform covariates selection.*

Description

choose2 select a model inside the multivariate scale mixtures of normal (MSMN), the multivariate scale mixtures of skew-normal (MSMSN), the multivariate skew scale mixtures of normal (MSSMN) or/and the multivariate scale mixtures of skew-normal-Cauchy (MSMSNC) classes. See details for supported distributions within each class. Then, implement the covariates selection based on the significance, the Akaike's information criteria (AIC) or Schwartz's information criteria (BIC).

Usage

```
choose2(y, X = NULL, max.iter = 1000, prec = 1e-04, class = "MSMN",
   est.var = TRUE, criteria = "AIC", criteria.cov = "AIC",
   significance = 0.05, cluster = FALSE)
```

Arguments

`y`	The multivariate vector of responses. The univariate case also is supported.
`X`	The regressor matrix.
`max.iter`	The maximum number of iterations.
`prec`	The convergence tolerance for parameters.
`class`	class in which will be performed a distribution: MSMN (default), MSSMN, MSMSN, MSMSNC or ALL (which consider all the mentioned classes). See details.
`est.var`	Logical. If TRUE the standard errors are estimated.
`criteria`	criteria to perform the selection model: AIC (default) or BIC.
`criteria.cov`	criteria to perform the covariates selection: AIC (default), BIC or significance.
`significance`	the level of significance to perform the covariate selection. Only used if criteria.cov="significance". By default is 0.05.
`cluster`	logical. If TRUE, parallel computing is used. FALSE is the default value.

Details

Supported models are:

In MSMN class: multivariate normal (MN), multivariate Student t (MT), multivariate slash (MSL), multivariate contaminated normal (MCN). See Lange and Sinsheimer (1993) for details.

In MSMSN class: multivariate skew-normal (MSN), multivariate skew-T (MSTT), multivariate skew-slash (MSSL2), multivariate skew-contaminated normal (MSCN2). See Zeller, Lachos and Vilca-Labra (2011) for details.

In MSSMN class: MSN, multivariate skew-t-normal (MSTN), multivariate skew-slash normal (MSSL), multivariate skew-contaminated normal (MSCN). See Louredo, Zeller and Ferreira (2021) for details.

In MSMSNC class: multivariate skew-normal-Cauchy (MSNC), multivariate skew-t-Expected-Cauchy (MSTEC), multivariate skew-slash-Expected-Cauchy (MSSLEC), multivariate skew-contaminated-Expected-Cauchy (MSCEC). See Kahrari et al. (2020) for details.

Note: the MSN distribution belongs to both, MSMSN and MSSMN classes.

Value

an object of class "skewMLRM" is returned. The object returned for this functions is a list containing the following components:

`coefficients`	A named vector of coefficients
`se`	A named vector of the standard errors for the estimated coefficients. Valid if est.var is TRUE and the hessian matrix is invertible.
`logLik`	The log-likelihood function evaluated in the estimated parameters for the selected model
`AIC`	Akaike's Information Criterion for the selected model
`BIC`	Bayesian's Information Criterion for the selected model
`iterations`	the number of iterations until convergence (if attached)
`conv`	An integer code for the selected model. 0 indicates successful completion. 1 otherwise.
`dist`	The distribution for which was performed the estimation.
`class`	The class for which was performed the estimation.
`function`	a string with the name of the used function.
`choose.crit`	the specified criteria to choose the distribution.
`choose.crit.cov`	the specified criteria to choose the covariates.
`y`	The multivariate vector of responses. The univariate case also is supported.
`X`	The regressor matrix (in a list form).
`fitted.models`	A vector with the fitted models
`selected.model`	Selected model based on the specified criteria.
`fitted.class`	Selected class based on the specified criteria.
`comment`	A comment indicating how many coefficients were eliminated

Author(s)

Clecio Ferreira, Diego Gallardo and Camila Zeller

References

Kahrari, F., Arellano-Valle, R.B., Ferreira, C.S., Gallardo, D.I. (2020) Some Simulation/computation in multivariate linear models of scale mixtures of skew-normal-Cauchy distributions. Communications in Statistics - Simulation and Computation. In press. DOI: 10.1080/03610918.2020.1804582

Lange, K., Sinsheimer, J.S. (1993). Normal/independent distributions and their applications in robust regression. Journal of Computational and Graphical Statistics 2, 175-198.

Louredo, G.M.S., Zeller, C.B., Ferreira, C.S. (2021). Estimation and influence diagnostics for the multivariate linear regression models with skew scale mixtures of normal distributions. Sankhya B. In press. DOI: 10.1007/s13571-021-00257-y

Zeller, C.B., Lachos, V.H., Vilca-Labra, F.E. (2011). Local influence analysis for regression models with scale mixtures of skew-normal distributions. Journal of Applied Statistics 38, 343-368.

Examples

```
data(ais, package="sn") ##Australian Institute of Sport data set
attach(ais)
##It is considered a bivariate regression model
##with Hg and SSF as response variables and
##Hc, Fe, Bfat and LBM as covariates
y<-cbind(Hg,SSF)
n<-nrow(y); m<-ncol(y)
X.aux=model.matrix(~Hc+Fe+Bfat+LBM)
p<-ncol(X.aux)
X<-array(0,dim=c(2*p,m,n))
for(i in 1:n) {
    X[1:p,1,i]=X.aux[i,,drop=FALSE]
    X[p+1:p,2,i]=X.aux[i,,drop=FALSE]
}
##See the covariate matrix X
##X

##Select a distribution within the MSMN class. Then, perform covariate
##selection based on the significance
fit.MSMN=choose2(y, X, class="MSMN")
summary(fit.MSMN)
##Identical process within the MSSMN class.
##may take some time on some systems
fit.MSSMN=choose2(y, X, class="MSSMN")
summary(fit.MSSMN)
```

chooseM *Choose a distribution in the MSMN, MSMSN, MSSMN and/or MSMSNC classes*

Description

choose.xxx select a model inside the xxx class, where xxx is the multivariate scale mixtures of normal (MSMN), the multivariate scale mixtures of skew-normal (MSMSN), the multivariate skew scale mixtures of normal (MSSMN) or the multivariate scale mixtures of skew-normal-Cauchy (MSMSNC) classes. See details for supported distributions within each class. choose.models select a model among the MSMN, MSMSN, MSSMN and MSMSNC classes.

Usage

```
choose.MSMN(y, X = NULL, max.iter = 1000, prec = 1e-4,
        est.var = TRUE, criteria = "AIC", cluster = FALSE)
choose.MSMSN(y, X = NULL, max.iter = 1000, prec = 1e-4,
        est.var = TRUE, criteria = "AIC", cluster = FALSE)
choose.MSSMN(y, X = NULL, max.iter = 1000, prec = 1e-4,
        est.var = TRUE, criteria = "AIC", cluster = FALSE)
choose.MSMSNC(y, X = NULL, max.iter = 1000, prec = 1e-4,
        est.var = TRUE, criteria = "AIC", cluster = FALSE)
choose.models(y, X = NULL, max.iter = 1000, prec = 1e-4,
        est.var = TRUE, criteria = "AIC", cluster = FALSE)
```

Arguments

y	The multivariate vector of responses. The univariate case also is supported.
X	The regressor matrix.
max.iter	The maximum number of iterations.
prec	The convergence tolerance for parameters.
est.var	Logical. If TRUE the standard errors are estimated.
criteria	criteria to perform the selection model: AIC (default) or BIC.
cluster	logical. If TRUE, parallel computing is used. FALSE is the default value.

Details

Supported models are:

In MSMN class: multivariate normal (MN), multivariate Student t (MT), multivariate slash (MSL), multivariate contaminated normal (MCN). See Lange and Sinsheimer (1993) for details.

In MSMSN class: multivariate skew-normal (MSN), multivariate skew-T (MSTT), multivariate skew-slash (MSSL2), multivariate skew-contaminated normal (MSCN2). See Zeller, Lachos and Vilca-Labra (2011) for details.

In MSSMN class: MSN, multivariate skew-t-normal (MSTN), multivariate skew-slash normal (MSSL), multivariate skew-contaminated normal (MSCN). See Louredo, Zeller and Ferreira (2021) for details.

In MSMSNC class: multivariate skew-normal-Cauchy (MSNC), multivariate skew-t-Expected-Cauchy (MSTEC), multivariate skew-slash-Expected-Cauchy (MSSLEC), multivariate skew-contaminated-Expected-Cauchy (MSCEC). See Kahrari et al. (2020) for details.

Note: the MSN distribution belongs to both, MSMSN and MSSMN classes.

Value

an object of class "skewMLRM" is returned. The object returned for this functions is a list containing the following components:

`coefficients`	A named vector of coefficients
`se`	A named vector of the standard errors for the estimated coefficients. Valid if est.var is TRUE and the hessian matrix is invertible.
`logLik`	The log-likelihood function evaluated in the estimated parameters for the selected model
`AIC`	Akaike's Information Criterion for the selected model
`BIC`	Bayesian's Information Criterion for the selected model
`iterations`	the number of iterations until convergence (if attached)
`conv`	An integer code for the selected model. 0 indicates successful completion. 1 otherwise.
`dist`	The distribution for which was performed the estimation.
`class`	The class for which was performed the estimation.
`function`	a string with the name of the used function.
`choose.crit`	the specified criteria to choose the distribution.
`y`	The multivariate vector of responses. The univariate case also is supported.
`X`	The regressor matrix (in a list form).
`fitted.models`	A vector with the fitted models
`selected.model`	Selected model based on the specified criteria.
`comment`	A comment indicating how many coefficients were eliminated

Note

This function does not consider selection of covariates.

Author(s)

Clecio Ferreira, Diego Gallardo and Camila Zeller

References

Kahrari, F., Arellano-Valle, R.B., Ferreira, C.S., Gallardo, D.I. (2020) Some Simulation/computation in multivariate linear models of scale mixtures of skew-normal-Cauchy distributions. Communications in Statistics - Simulation and Computation. In press. DOI: 10.1080/03610918.2020.1804582

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Zeller, C.B., Lachos, V.H., Vilca-Labra, F.E. (2011). Local influence analysis for regression models with scale mixtures of skew-normal distributions. Journal of Applied Statistics 38, 343-368.

Examples

```
data(ais, package="sn") ##Australian Institute of Sport data set
attach(ais)
##It is considered a bivariate regression model
##with Hg and SSF as response variables and
##Hc, Fe, Bfat and LBM as covariates
y<-cbind(Hg,SSF)
n<-nrow(y); m<-ncol(y)
X.aux=model.matrix(~Hc+Fe+Bfat+LBM)
p<-ncol(X.aux)
X<-array(0,dim=c(2*p,m,n))
for(i in 1:n) {
    X[1:p,1,i]=X.aux[i,,drop=FALSE]
    X[p+1:p,2,i]=X.aux[i,,drop=FALSE]
}
##See the covariate matrix X
##X
##Select a distribution within the MSMN class.

fit.MSMN=choose.MSMN(y,X)
summary(fit.MSMN)
##Identical process within the MSSMN class.
##may take some time on some systems
fit.MSSMN=choose.MSSMN(y,X)
summary(fit.MSSMN)
```

`distMahal` *Mahalanobis distance for fitted models in the MSMN, MSMSN, MSSMN and MSMSNC classes*

Description

Compute and plot the Mahalanobis distance for any supported model in the multivariate scale mixtures of normal (MSMN), multivariate scale mixtures of skew-normal (MSMSN), multivariate skew scale mixtures of normal (MSSMN) or multivariate scale mixtures of skew-normal-Cauchy (MSMSNC) classes. See details for supported distributions.

Usage

```
distMahal(object, alpha = 0.95, ...)
```

Arguments

`object`	an object of class "skewMLRM" returned by one of the following functions: estimate.xxx, choose.yyy, choose2, mbackcrit or mbacksign. See details for supported distributions.
`alpha`	significance level (0.05 by default).
`...`	aditional graphical parameters

Details

Supported models are:

In MSMN class: multivariate normal (MN), multivariate Student t (MT), multivariate slash (MSL), multivariate contaminated normal (MCN). See Lange and Sinsheimer (1993) for details.

In MSMSN class: multivariate skew-normal (MSN), multivariate skew-T (MSTT), multivariate skew-slash (MSSL2), multivariate skew-contaminated normal (MSCN2). See Zeller, Lachos and Vilca-Labra (2011) for details.

In MSSMN class: MSN, multivariate skew-t-normal (MSTN), multivariate skew-slash normal (MSSL), multivariate skew-contaminated normal (MSCN). See Louredo, Zeller and Ferreira (2021) for details.

In MSMSNC class: multivariate skew-normal-Cauchy (MSNC), multivariate skew-t-Expected-Cauchy (MSTEC), multivariate skew-slash-Expected-Cauchy (MSSLEC), multivariate skew-contaminated-Expected-Cauchy (MSCEC). See Kahrari et al. (2020) for details.

Note: the MSN distribution belongs to both, MSMSN and MSSMN classes.

Value

distMahal provides an object of class skewMLRM related to compute the Mahalanobis distance for all the observations and a cut-off to detect possible influent observations based on the specified significance (0.05 by default).

an object of class "skewMLRM" is returned. The object returned for this functions is a list containing the following components:

`Mahal`	the Mahalanobis distance for all the observations
`function`	a string with the name of the used function.
`dist`	The distribution for which was performed the estimation.
`class`	The class for which was performed the estimation.

alpha	specified level of significance (0.05 by default).
cut	the cut-off to detect possible influent observations based on the specified significance.
y	The multivariate vector of responses. The univariate case also is supported.
X	The regressor matrix (in a list form).

Author(s)

Clecio Ferreira, Diego Gallardo and Camila Zeller

References

Kahrari, F., Arellano-Valle, R.B., Ferreira, C.S., Gallardo, D.I. (2020) Some Simulation/computation in multivariate linear models of scale mixtures of skew-normal-Cauchy distributions. Communications in Statistics - Simulation and Computation. In press. DOI: 10.1080/03610918.2020.1804582

Lange, K., Sinsheimer, J.S. (1993). Normal/independent distributions and their applications in robust regression. Journal of Computational and Graphical Statistics 2, 175-198.

Louredo, G.M.S., Zeller, C.B., Ferreira, C.S. (2021). Estimation and influence diagnostics for the multivariate linear regression models with skew scale mixtures of normal distributions. Sankhya B. In press. DOI: 10.1007/s13571-021-00257-y

Zeller, C.B., Lachos, V.H., Vilca-Labra, F.E. (2011). Local influence analysis for regression models with scale mixtures of skew-normal distributions. Journal of Applied Statistics 38, 343-368.

Examples

```
set.seed(2020)
n=200   # length of the sample
nv<-3   # number of explanatory variables
p<-nv+1 # nv + intercept
m<-4    # dimension of Y
q0=p*m
X<-array(0,c(q0,m,n))
for(i in 1:n) {
    aux=rep(1,p)
    aux[2:p]<-rMN(1,mu=rnorm(nv),Sigma=diag(nv)) ##simulating covariates
    mi=matrix(0,q0,m)
    for (j in 1:m) mi[((j-1)*p+1):(j*p),j]=aux
    X[,,i]<-mi
} ##X is the simulated regressor matrix
betas<-matrix(rnorm(q0),ncol=1) ##True betas
Sigmas <- clusterGeneration::genPositiveDefMat(m,rangeVar=c(1,3),
lambdaLow=1, ratioLambda=3)$Sigma ##True Sigma
y=matrix(0,n,m)
for(i in 1:n) {
     mui<-t(X[,,i])%*%betas
     y[i,]<-rMN(n=1,c(mui),Sigmas) ## simulating the response vector
}
fit.MN=estimate.MN(y,X)        #fit the MN model
mahal.MN=distMahal(fit.MN)     #compute the Mahalanobis distances for MN model
```

```
plot(mahal.MN)              #plot the Mahalanobis distances for MN model
mahal.MN$Mahal             #presents the Malahanobis distances
```

estimateM *Fitting a model in the MSMN, MSMSN, MSSMN and MSMSNC classes*

Description

estimate.Mxxx computes the maximum likelihood estimates for the distribution xxx, where xxx is any supported model in the multivariate scale mixtures of normal (MSMN), multivariate scale mixtures of skew-normal (MSMSN), multivariate skew scale mixtures of normal (MSSMN) or multivariate scale mixtures of skew-normal-Cauchy (MSMSNC) classes. See details for supported distributions.

Usage

```
estimate.MN(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE)
estimate.MT(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE,
     nu.min = 2.0001)
estimate.MSL(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE,
     nu.min = 2.0001)
estimate.MCN(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE)
estimate.MSN(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE)
estimate.MSTN(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE,
     nu.min = 2.0001)
estimate.MSSL(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE,
     nu.min = 2.0001)
estimate.MSCN(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE)
estimate.MSTT(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE,
     nu.fixed = 3, nu.min = 2.0001)
estimate.MSSL2(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE,
     nu.fixed = 3, nu.min = 2.0001)
estimate.MSCN2(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE,
      nu.fixed = 0.5, gamma.fixed = 0.5)
estimate.MSNC(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE)
estimate.MSTEC(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE,
      nu.fixed = 3, nu.min = 2.0001)
estimate.MSSLEC(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE,
      nu.fixed = 3, nu.min = 2.0001)
estimate.MSCEC(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE,
      nu.fixed = 0.5, gamma.fixed = 0.5)
```

Arguments

y	The multivariate vector of responses. The univariate case also is supported.
X	The regressor matrix.
max.iter	The maximum number of iterations.

prec	The convergence tolerance for parameters.
est.var	Logical. If TRUE the standard errors are estimated.
nu.fixed	If a numerical value is provided, the estimation consider nu as fixed. To estimate nu, use nu.fixed=FALSE. Avaliable for MSTT, MSSL2, MSCN2, MSTEC, MSSLEC and MSCEC distributions. For MSTT, MSSL2, MSTEC and MSSLEC, the default value is 3 and nu should be greater than 1. For MSCN2 and MSCEC, the default value is 0.5 and nu should be in the (0,1) interval.
gamma.fixed	If a numerical value is provided, the estimation consider gamma as fixed. To estimate gamma, use gamma.fixed=FALSE. Avaliable for MSCN2 and MSCEC distributions. For MSCN2 and MSCEC, the default value is 0.5 and gamma should be in the (0,1) interval.
nu.min	Lower value to perform the maximization for nu. For MSTT, MSSL2, MSTEC and MSSLEC is used only when nu.fixed=FALSE.

Details

Supported models are:

In MSMN class: multivariate normal (MN), multivariate Student t (MT), multivariate slash (MSL), multivariate contaminated normal (MCN). See Lange and Sinsheimer (1993) for details.

In MSMSN class: multivariate skew-normal (MSN), multivariate skew-T (MSTT), multivariate skew-slash (MSSL2), multivariate skew-contaminated normal (MSCN2). See Zeller, Lachos and Vilca-Labra (2011) for details.

In MSSMN class: MSN, multivariate skew-t-normal (MSTN), multivariate skew-slash normal (MSSL), multivariate skew-contaminated normal (MSCN). See Louredo, Zeller and Ferreira (2021) for details.

In MSMSNC class: multivariate skew-normal-Cauchy (MSNC), multivariate skew-t-Expected-Cauchy (MSTEC), multivariate skew-slash-Expected-Cauchy (MSSLEC), multivariate skew-contaminated-Expected-Cauchy (MSCEC). See Kahrari et al. (2020) for details.

Note: the MSN distribution belongs to both, MSMSN and MSSMN classes.

Value

an object of class "skewMLRM" is returned. The object returned for this functions is a list containing the following components:

coefficients	A named vector of coefficients
se	A named vector of the standard errors for the estimated coefficients. Valid if est.var is TRUE and the hessian matrix is invertible.
nu	The estimated or fixed nu (only for MSTT, MSSL2, MSCN2, MSTEC, MSSLEC and MSCEC models)
gamma	The estimated or fixed gamma (only for MSCN2 and MSCEC models)
logLik	The log-likelihood function evaluated in the estimated parameters
AIC	Akaike's Information Criterion
BIC	Bayesian's Information Criterion

iterations	the number of iterations until convergence (if attached)
time	execution time in seconds
conv	An integer code. 0 indicates successful completion. 1 otherwise.
dist	The distribution for which was performed the estimation.
class	The class for which was performed the estimation.
n	The sample size
y	The multivariate vector of responses. The univariate case also is supported.
X	The regressor matrix (in a list form).
function	a string with the name of the used function.

Note

In MT, MSL, MSTN, MSSL, MSTT, MSSL2, MSTEC and MSSLEC distributions, nu>2 guarantees that the mean and variance exist, nu>1 guarantees the existence only for the mean and for nu<=1, the mean and variance of the distribution is not finite.

Author(s)

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References

Kahrari, F., Arellano-Valle, R.B., Ferreira, C.S., Gallardo, D.I. (2020) Some Simulation/computation in multivariate linear models of scale mixtures of skew-normal-Cauchy distributions. Communications in Statistics - Simulation and Computation. In press. DOI: 10.1080/03610918.2020.1804582

Lange, K., Sinsheimer, J.S. (1993). Normal/independent distributions and their applications in robust regression. Journal of Computational and Graphical Statistics 2, 175-198.

Louredo, G.M.S., Zeller, C.B., Ferreira, C.S. (2021). Estimation and influence diagnostics for the multivariate linear regression models with skew scale mixtures of normal distributions. Sankhya B. In press. DOI: 10.1007/s13571-021-00257-y

Zeller, C.B., Lachos, V.H., Vilca-Labra, F.E. (2011). Local influence analysis for regression models with scale mixtures of skew-normal distributions. Journal of Applied Statistics 38, 343-368.

Examples

```
data(ais, package="sn") ##Australian Institute of Sport data set
attach(ais)
##It is considered a bivariate regression model
##with Hg and SSF as response variables and
##Hc, Fe, Bfat and LBM as covariates
y<-cbind(Hg,SSF)
n<-nrow(y); m<-ncol(y)
X.aux=model.matrix(~Hc+Fe+Bfat+LBM)
p<-ncol(X.aux)
X<-array(0,dim=c(2*p,m,n))
for(i in 1:n) {
    X[1:p,1,i]=X.aux[i,,drop=FALSE]
```

```
    X[p+1:p,2,i]=X.aux[i,,drop=FALSE]
}
##See the covariate matrix X
##X
fit.MN=estimate.MN(y, X) ##Estimate the parameters for the MN regression model
summary(fit.MN)
fit.MT=estimate.MT(y, X) ##Estimate the parameters for the MT regression model
summary(fit.MT)

##may take some time on some systems
fit.MSSL=estimate.MSSL(y, X)   ##Estimate the parameters for the MSSL regression model
summary(fit.MSSL)
fit.MSTT=estimate.MSTT(y, X)   ##Estimate the parameters for the MSTT regression model
summary(fit.MSTT)
fit.MSNC=estimate.MSNC(y, X)   ##Estimate the parameters for the MSNC regression model
summary(fit.MSNC)
fit.MSCEC=estimate.MSCEC(y, X) ##Estimate the parameters for the MSCEC regression model
summary(fit.MSCEC)
```

`FIM` *Observed Fisher information matrix for distributions in the MSMN, MSMSN, MSSMN and MSMSNC classes.*

Description

FI.xxx computes the observed Fisher information (FI) matrix for the distribution xxx, where xxx is any supported model in the multivariate scale mixtures of normal (MSMN), multivariate scale mixtures of skew-normal (MSMSN), multivariate skew scale mixtures of normal (MSSMN) or multivariate scale mixtures of skew-normal-Cauchy (MSMSNC) classes. See details for supported distributions.

Usage

```
FI.MN(P, y, X)
FI.MT(P, y, X)
FI.MSL(P, y, X)
FI.MCN(P, y, X)
FI.MSN(P, y, X)
FI.MSTN(P, y, X)
FI.MSSL(P, y, X)
FI.MSCN(P, y, X)
FI.MSTT(P, y, X, nu)
FI.MSSL2(P, y, X, nu)
FI.MSCN2(P, y, X, nu, gamma)
FI.MSNC(P, y, X)
FI.MSTEC(P, y, X, nu)
FI.MSSLEC(P, y, X, nu)
FI.MSCEC(P, y, X, nu, gamma)
```

Arguments

`P`	the estimated parameters returned by a function of the form estimate.xxx, where xxx is a supported distribution.
`y`	The multivariate vector of responses. The univariate case also is supported.
`X`	The regressor matrix.
`nu`	nu parameter. Only for MSTT, MSSL2, MSTEC, MSSLEC and MSCEC distributions.
`gamma`	gamma parameter. Only for MSCN2 and MSCEC distributions.

Details

Supported models are:

In MSMN class: multivariate normal (MN), multivariate Student t (MT), multivariate slash (MSL), multivariate contaminated normal (MCN). See Lange and Sinsheimer (1993) for details.

In MSMSN class: multivariate skew-normal (MSN), multivariate skew-T (MSTT), multivariate skew-slash (MSSL2), multivariate skew-contaminated normal (MSCN2). See Zeller, Lachos and Vilca-Labra (2011) for details.

In MSSMN class: MSN, multivariate skew-t-normal (MSTN), multivariate skew-slash normal (MSSL), multivariate skew-contaminated normal (MSCN). See Louredo, Zeller and Ferreira (2021) for details.

In MSMSNC class: multivariate skew-normal-Cauchy (MSNC), multivariate skew-t-Expected-Cauchy (MSTEC), multivariate skew-slash-Expected-Cauchy (MSSLEC), multivariate skew-contaminated-Expected-Cauchy (MSCEC). See Kahrari et al. (2020) for details.

Note: the MSN distribution belongs to both, MSMSN and MSSMN classes.

Value

A matrix with the observed FI matrix for the specified model.

Note

For MSTEC and MSSLEC and distributions, nu>0 is considered as fixed. For MSCEC distribution, 0<nu<1 and 0<gamma<1 are considered as fixed.

Author(s)

Clecio Ferreira, Diego Gallardo and Camila Zeller.

References

Kahrari, F., Arellano-Valle, R.B., Ferreira, C.S., Gallardo, D.I. (2020) Some Simulation/computation in multivariate linear models of scale mixtures of skew-normal-Cauchy distributions. Communications in Statistics - Simulation and Computation. In press. DOI: 10.1080/03610918.2020.1804582

Lange, K., Sinsheimer, J.S. (1993). Normal/independent distributions and their applications in robust regression. Journal of Computational and Graphical Statistics 2, 175-198.

Louredo, G.M.S., Zeller, C.B., Ferreira, C.S. (2021). Estimation and influence diagnostics for the multivariate linear regression models with skew scale mixtures of normal distributions. Sankhya B. In press. DOI: 10.1007/s13571-021-00257-y

Zeller, C.B., Lachos, V.H., Vilca-Labra, F.E. (2011). Local influence analysis for regression models with scale mixtures of skew-normal distributions. Journal of Applied Statistics 38, 343-368.

Examples

```
set.seed(2020)
n=200   # length of the sample
nv<-3   # number of explanatory variables
p<-nv+1 # nv + intercept
m<-4    # dimension of Y
q0=p*m
X<-array(0,c(q0,m,n))
for(i in 1:n) {
    aux=rep(1,p)
    aux[2:p]<-rMN(1,mu=rnorm(nv),Sigma=diag(nv))
    mi=matrix(0,q0,m)
    for (j in 1:m) mi[((j-1)*p+1):(j*p),j]=aux
    X[,,i]<-mi
} #Simulated matrix covariates
betas<-matrix(rnorm(q0),ncol=1) ## True betas
Sigmas <- clusterGeneration::genPositiveDefMat(m,rangeVar=c(1,3),
lambdaLow=1, ratioLambda=3)$Sigma ##True Sigma
lambda<-rnorm(m) ##True lambda
y=matrix(0,n,m)
for(i in 1:n) {
     mui<-t(X[,,i])%*%betas
     y[i,]<-rMSN(n=1,c(mui),Sigmas,lambda)}

fit.MSN=estimate.MSN(y,X) ##Estimate parameters for MSN model
fit.MSN  ## Output of estimate.MSN
summary(fit.MSN)
fit.MSN$se ##Estimated standard errors by the estimate.MSN function
##Estimated standard errors by minus the square root of
##the diagonal from the observed FI matrix of the MSN model
sqrt(diag(solve(-FI.MSN(fit.MSN$coefficients, y, X))))
```

---

matrix.sqrt *Square root of a matrix*

Description

Compute the square root of a matrix

Usage

```
matrix.sqrt(A)
```

Arguments

A	a symmetric semi-definite positive matrix

Value

A symmetric matrix, say B, such as B^t*B=A

Note

For internal use.

Author(s)

Clecio Ferreira, Diego Gallardo and Camila Zeller.

Examples

```
A<-matrix(c(1,2,2,5),nrow=2)
B<-matrix.sqrt(A)
##Recovering A
t(B)%*%B
A
```

---

mbackcrit *Multivariate backward based on the AIC or BIC criteria*

Description

mbackcrit implements the covariates selection based on backward and the Akaike's information criteria (AIC) or Schwartz's information criteria (BIC) in a specified multivariate model in the multivariate scale mixtures of normal (MSMN), multivariate scale mixtures of skew-normal (MSMSN), multivariate skew scale mixtures of normal (MSSMN) or multivariate scale mixtures of skew-normal-Cauchy (MSMSNC) classes. See details for avaliable distributions.

Usage

```
mbackcrit(y, X = NULL, max.iter = 1000, prec = 1e-04, dist = "MN",
criteria = "AIC", est.var=TRUE, cluster = FALSE, ...)
```

Arguments

y	The multivariate vector of responses. The univariate case also is supported.
X	The regressor matrix. It should include intercept term for all the variates.
max.iter	The maximum number of iterations.
prec	The convergence tolerance for parameters.
dist	the multivariate distribution in which the covariates selection will be implemented.

`criteria`	criteria used to perform the covariates selection. AIC (default) and BIC available.
`est.var`	Logical. If TRUE the standard errors are estimated.
`cluster`	logical. If TRUE, parallel computing is used. FALSE is the default value.
`...`	Possible aditional arguments. For instance, for MSTT, MSSL2, MSTEC and MSSLEC distributions should be added nu.min and nu.fixed related to specifications for the nu parameter.

Details

Supported models are:

In MSMN class: multivariate normal (MN), multivariate Student t (MT), multivariate slash (MSL), multivariate contaminated normal (MCN). See Lange and Sinsheimer (1993) for details.

In MSMSN class: multivariate skew-normal (MSN), multivariate skew-T (MSTT), multivariate skew-slash (MSSL2), multivariate skew-contaminated normal (MSCN2). See Zeller, Lachos and Vilca-Labra (2011) for details.

In MSSMN class: MSN, multivariate skew-t-normal (MSTN), multivariate skew-slash normal (MSSL), multivariate skew-contaminated normal (MSCN). See Louredo, Zeller and Ferreira (2021) for details.

In MSMSNC class: multivariate skew-normal-Cauchy (MSNC), multivariate skew-t-Expected-Cauchy (MSTEC), multivariate skew-slash-Expected-Cauchy (MSSLEC), multivariate skew-contaminated-Expected-Cauchy (MSCEC). See Kahrari et al. (2020) for details.

Note: the MSN distribution belongs to both, MSMSN and MSSMN classes.

Value

an object of class "skewMLRM" is returned. The object returned for this functions is a list containing the following components:

`coefficients`	A named vector of coefficients
`se`	A named vector of the standard errors for the estimated coefficients. Valid if est.var is TRUE and the hessian matrix is invertible.
`logLik`	The log-likelihood function evaluated in the estimated parameters for the selected model
`AIC`	Akaike's Information Criterion for the selected model
`BIC`	Bayesian's Information Criterion for the selected model
`iterations`	the number of iterations until convergence (if attached)
`conv`	An integer code for the selected model. 0 indicates successful completion. 1 otherwise.
`dist`	The distribution for which was performed the estimation.
`class`	The class for which was performed the estimation.
`choose.crit`	the specified criteria to choose the distribution.
`comment`	A comment indicating how many coefficients were eliminated

y	The multivariate vector of responses. The univariate case also is supported.
X	The regressor matrix (in a list form).
function	a string with the name of the used function.

Author(s)

Clecio Ferreira, Diego Gallardo and Camila Zeller.

References

Kahrari, F., Arellano-Valle, R.B., Ferreira, C.S., Gallardo, D.I. (2020) Some Simulation/computation in multivariate linear models of scale mixtures of skew-normal-Cauchy distributions. Communications in Statistics - Simulation and Computation. In press. DOI: 10.1080/03610918.2020.1804582

Lange, K., Sinsheimer, J.S. (1993). Normal/independent distributions and their applications in robust regression. Journal of Computational and Graphical Statistics 2, 175-198.

Louredo, G.M.S., Zeller, C.B., Ferreira, C.S. (2021). Estimation and influence diagnostics for the multivariate linear regression models with skew scale mixtures of normal distributions. Sankhya B. In press. DOI: 10.1007/s13571-021-00257-y

Zeller, C.B., Lachos, V.H., Vilca-Labra, F.E. (2011). Local influence analysis for regression models with scale mixtures of skew-normal distributions. Journal of Applied Statistics 38, 343-368.

Examples

```
data(ais, package="sn") ##Australian Institute of Sport data set
attach(ais)
##It is considered a bivariate regression model
##with Hg and SSF as response variables and
##Hc, Fe, Bfat and LBM as covariates
y<-cbind(Hg,SSF)
n<-nrow(y); m<-ncol(y)
X.aux=model.matrix(~Hc+Fe+Bfat+LBM)
p<-ncol(X.aux)
X<-array(0,dim=c(2*p,m,n))
for(i in 1:n) {
    X[1:p,1,i]=X.aux[i,,drop=FALSE]
    X[p+1:p,2,i]=X.aux[i,,drop=FALSE]
}
##See the regressor matrix X
##X

##Perform covariates selection in the MN distribution
##based on the AIC criteria
##may take some time on some systems
fit.MN=mbackcrit(y, X, dist="MN")
summary(fit.MN)
##Identical process for MT distribution
fit.MT=mbackcrit(y, X, dist="MT")
summary(fit.MT)
##and for MSN distribution
fit.MSN=mbackcrit(y, X, dist="MSN")
```

```
summary(fit.MSN)
```

---

mbacksign *Multivariate Backward Based on Significance*

Description

mbacksign implements the covariates selection based on the significance of the covariates in a specified multivariate model in the multivariate scale mixtures of normal (MSMN), multivariate scale mixtures of skew-normal (MSMSN), multivariate skew scale mixtures of normal (MSSMN) or multivariate scale mixtures of skew-normal-Cauchy (MSMSNC) classes. See details for avaliable distributions.

Usage

```
mbacksign(y, X = NULL, max.iter = 1000, prec = 1e-04, dist = "MN",
     significance = 0.05, ...)
```

Arguments

`y`	The multivariate vector of responses. The univariate case also is supported.
`X`	The regressor matrix. It should include intercept term for all the variates.
`max.iter`	The maximum number of iterations.
`prec`	The convergence tolerance for parameters.
`dist`	the multivariate distribution in which the covariates selection will be implemented.
`significance`	the level of significance to perform the covariate selection. By default is 0.05.
`...`	Possible aditional arguments. For instance, for MSTT, MSSL2, MSTEC and MSSLEC distributions should be added nu.min and nu.fixed related to specifications for the nu parameter.

Details

Supported models are:

In MSMN class: multivariate normal (MN), multivariate Student t (MT), multivariate slash (MSL), multivariate contaminated normal (MCN). See Lange and Sinsheimer (1993) for details.

In MSMSN class: multivariate skew-normal (MSN), multivariate skew-T (MSTT), multivariate skew-slash (MSSL2), multivariate skew-contaminated normal (MSCN2). See Zeller, Lachos and Vilca-Labra (2011) for details.

In MSSMN class: MSN, multivariate skew-t-normal (MSTN), multivariate skew-slash normal (MSSL), multivariate skew-contaminated normal (MSCN). See Louredo, Zeller and Ferreira (2021) for details.

In MSMSNC class: multivariate skew-normal-Cauchy (MSNC), multivariate skew-t-Expected-Cauchy (MSTEC), multivariate skew-slash-Expected-Cauchy (MSSLEC), multivariate skew-contaminated-Expected-Cauchy (MSCEC). See Kahrari et al. (2020) for details.

Note: the MSN distribution belongs to both, MSMSN and MSSMN classes.

Value

an object of class "skewMLRM" is returned. The object returned for this functions is a list containing the following components:

- `coefficients` A named vector of coefficients
- `se` A named vector of the standard errors for the estimated coefficients. Valid if est.var is TRUE and the hessian matrix is invertible.
- `logLik` The log-likelihood function evaluated in the estimated parameters for the selected model
- `AIC` Akaike's Information Criterion for the selected model
- `BIC` Bayesian's Information Criterion for the selected model
- `iterations` the number of iterations until convergence (if attached)
- `conv` An integer code for the selected model. 0 indicates successful completion. 1 otherwise.
- `dist` The distribution for which was performed the estimation.
- `class` The class for which was performed the estimation.
- `choose.crit` the specified criteria to choose the distribution.
- `comment` A comment indicating how many coefficients were eliminated
- `eliminated` An string vector with the eliminated betas (in order of elimination).
- `y` The multivariate vector of responses. The univariate case also is supported.
- `X` The regressor matrix (in a list form).
- `significance` The specified level of significance (0.05 by default).
- `function` a string with the name of the used function.

Author(s)

Clecio Ferreira, Diego Gallardo and Camila Zeller.

References

Kahrari, F., Arellano-Valle, R.B., Ferreira, C.S., Gallardo, D.I. (2020) Some Simulation/computation in multivariate linear models of scale mixtures of skew-normal-Cauchy distributions. Communications in Statistics - Simulation and Computation. In press. DOI: 10.1080/03610918.2020.1804582

Lange, K., Sinsheimer, J.S. (1993). Normal/independent distributions and their applications in robust regression. Journal of Computational and Graphical Statistics 2, 175-198.

Louredo, G.M.S., Zeller, C.B., Ferreira, C.S. (2021). Estimation and influence diagnostics for the multivariate linear regression models with skew scale mixtures of normal distributions. Sankhya B. In press. DOI: 10.1007/s13571-021-00257-y

Zeller, C.B., Lachos, V.H., Vilca-Labra, F.E. (2011). Local influence analysis for regression models with scale mixtures of skew-normal distributions. Journal of Applied Statistics 38, 343-368.

Examples

```
data(ais, package="sn") ##Australian Institute of Sport data set
attach(ais)
##It is considered a bivariate regression model
##with Hg and SSF as response variables and
##Hc, Fe, Bfat and LBM as covariates
y<-cbind(Hg,SSF)
n<-nrow(y); m<-ncol(y)
X.aux=model.matrix(~Hc+Fe+Bfat+LBM)
p<-ncol(X.aux)
X<-array(0,dim=c(2*p,m,n))
for(i in 1:n) {
    X[1:p,1,i]=X.aux[i,,drop=FALSE]
    X[p+1:p,2,i]=X.aux[i,,drop=FALSE]
}
##See the regressor matrix X
##X
##Perform covariates selection in the MN distribution
##based on a significance level of 1%, 5% and 10%

##may take some time on some systems
fit.MN.01=mbacksign(y, X, dist="MN", sign=0.01)
fit.MN.05=mbacksign(y, X, dist="MN", sign=0.05)
fit.MN.10=mbacksign(y, X, dist="MN", sign=0.10)
summary(fit.MN.01)
summary(fit.MN.05)
summary(fit.MN.10)
##identical process in the MCN model with
##significance level of 5%
fit.MCN=mbacksign(y, X, dist="MCN")
summary(fit.MCN)
##for MSSL model
fit.MSSL=mbacksign(y, X, dist="MSSL")
summary(fit.MSSL)
##for MSNC model
fit.MSNC=mbacksign(y, X, dist="MSNC")
summary(fit.MSNC)
```

`plot.skewMLRM` *Plot an object of the "skewMLRM" class produced with the function distMahal.*

Description

Plot the Mahalanobis distance for a object of the class "skewMLRM" produced by the function distMahal.

Usage

```
## S3 method for class 'skewMLRM'
plot(x, ...)
```

Arguments

x	an object of the class "skewMLRM" produced by the function distMahal.
...	for graphical extra arguments

Details

Supported models are:

In MSMN class: multivariate normal (MN), multivariate Student t (MT), multivariate slash (MSL), multivariate contaminated normal (MCN). See Lange and Sinsheimer (1993) for details.

In MSMSN class: multivariate skew-normal (MSN), multivariate skew-T (MSTT), multivariate skew-slash (MSSL2), multivariate skew-contaminated normal (MSCN2). See Zeller, Lachos and Vilca-Labra (2011) for details.

In MSSMN class: MSN, multivariate skew-t-normal (MSTN), multivariate skew-slash normal (MSSL), multivariate skew-contaminated normal (MSCN). See Louredo, Zeller and Ferreira (2021) for details.

In MSMSNC class: multivariate skew-normal-Cauchy (MSNC), multivariate skew-t-Expected-Cauchy (MSTEC), multivariate skew-slash-Expected-Cauchy (MSSLEC), multivariate skew-contaminated-Expected-Cauchy (MSCEC). See Kahrari et al. (2020) for details.

Note: the MSN distribution belongs to both, MSMSN and MSSMN classes.

The functions which generate an object of the class "skewMLRM" are

estimate.xxx: where xxx can be MN, MT, MSL, MCN, MSN, MSTN, MSSL, MSCN, MSTT, MSSL2, MSCN2, MSNC, MSTEC, MSSLEC or MSCEC.

choose.yyy: where yyy can be MSMN, MSSMN, MSMSN, MSMSNC or models.

chose2, mbackcrit and mbacksign.

Value

A complete summary for the coefficients extracted from a skewMLRM object.

Author(s)

Clecio Ferreira, Diego Gallardo and Camila Zeller

References

Kahrari, F., Arellano-Valle, R.B., Ferreira, C.S., Gallardo, D.I. (2020) Some Simulation/computation in multivariate linear models of scale mixtures of skew-normal-Cauchy distributions. Communications in Statistics - Simulation and Computation. In press. DOI: 10.1080/03610918.2020.1804582

Lange, K., Sinsheimer, J.S. (1993). Normal/independent distributions and their applications in robust regression. Journal of Computational and Graphical Statistics 2, 175-198.

Louredo, G.M.S., Zeller, C.B., Ferreira, C.S. (2021). Estimation and influence diagnostics for the multivariate linear regression models with skew scale mixtures of normal distributions. Sankhya B. In press. DOI: 10.1007/s13571-021-00257-y

Zeller, C.B., Lachos, V.H., Vilca-Labra, F.E. (2011). Local influence analysis for regression models with scale mixtures of skew-normal distributions. Journal of Applied Statistics 38, 343-368.

Examples

```
data(ais, package="sn") ##Australian Institute of Sport data set
attach(ais)
##It is considered a bivariate regression model
##with Hg and SSF as response variables and
##Hc, Fe, Bfat and LBM as covariates
y<-cbind(Hg,SSF)
n<-nrow(y); m<-ncol(y)
X.aux=model.matrix(~Hc+Fe+Bfat+LBM)
p<-ncol(X.aux)
X<-array(0,dim=c(2*p,m,n))
for(i in 1:n) {
    X[1:p,1,i]=X.aux[i,,drop=FALSE]
    X[p+1:p,2,i]=X.aux[i,,drop=FALSE]
}
##See the covariate matrix X
##X

fit.MN=estimate.MN(y, X)   #Fit the MN distribution
res.MN=distMahal(fit.MN)   #Compute the Mahalanobis distances
plot(res.MN)               #Plot the Mahalanobis distances
#
fit.MSN=estimate.MSN(y, X)  #Fit the MSN distribution
res.MSN=distMahal(fit.MSN)  #Compute the Mahalanobis distances
plot(res.MSN)               #Plot the Mahalanobis distances
```

`rM` *Random generation for models in the MSMN, MSMSN, MSSMN and MSMSNC classes*

Description

rxxx generates random values for the distribution xxx, where xxx is any supported model in the multivariate scale mixtures of normal (MSMN), multivariate scale mixtures of skew-normal (MSMSN), multivariate skew scale mixtures of normal (MSSMN) or multivariate scale mixtures of skew-normal-Cauchy (MSMSNC) classes. See details for supported distributions.

Usage

```
rMN(n, mu, Sigma)
rMT(n, mu, Sigma, nu = 1)
```

```
rMSL(n, mu, Sigma, nu = 1)
rMCN(n, mu, Sigma, nu = 0.5, gamma = 0.5)
rMSN(n, mu, Sigma, lambda)
rMSTN(n, mu, Sigma, lambda, nu = 1)
rMSSL(n, mu, Sigma, lambda, nu = 1)
rMSCN(n, mu, Sigma, lambda, nu = 0.5, gamma = 0.5)
rMSTT(n, mu, Sigma, lambda, nu = 1)
rMSSL2(n, mu, Sigma, lambda, nu = 1)
rMSCN2(n, mu, Sigma, lambda, nu = 0.5, gamma = 0.5)
rMSNC(n, mu, Sigma, lambda)
rMSTEC(n, mu, Sigma, lambda, nu = 1)
rMSSLEC(n, mu, Sigma, lambda, nu = 1)
rMSCEC(n, mu, Sigma, lambda, nu = 0.5, gamma = 0.5)
```

Arguments

`n`	number of observations to be generated.
`mu`	vector of location parameters.
`Sigma`	covariance matrix (a positive definite matrix).
`lambda`	vector of shape parameters.
`nu`	nu parameter. A positive scalar for MT, MSL, MSTN, MSSL, MSTT, MSSL2, MSTEC and MSSLEC models. A value in the interval (0,1) for MCN, MSCN, MSCN2 and MSCEC models.
`gamma`	gamma parameter. A value in the interval (0,1) for MCN, MSCN, MSCN2 and MSCEC models.

Details

Supported models are:

In MSMN class: multivariate normal (MN), multivariate Student t (MT), multivariate slash (MSL), multivariate contaminated normal (MCN). See Lange and Sinsheimer (1993) for details.

In MSMSN class: multivariate skew-normal (MSN), multivariate skew-T (MSTT), multivariate skew-slash (MSSL2), multivariate skew-contaminated normal (MSCN2). See Zeller, Lachos and Vilca-Labra (2011) for details.

In MSSMN class: MSN, multivariate skew-t-normal (MSTN), multivariate skew-slash normal (MSSL), multivariate skew-contaminated normal (MSCN). See Louredo, Zeller and Ferreira (2021) for details.

In MSMSNC class: multivariate skew-normal-Cauchy (MSNC), multivariate skew-t-Expected-Cauchy (MSTEC), multivariate skew-slash-Expected-Cauchy (MSSLEC), multivariate skew-contaminated-Expected-Cauchy (MSCEC). See Kahrari et al. (2020) for details.

Note: the MSN distribution belongs to both, MSMSN and MSSMN classes.

MN used `mvrnorm`. For MT, MSL and MCN, the generation is based on the MSMN class. See Lange and Sinsheimer (1993) for details. For MSTN, MSSL and MSCN, the generation is based on the MSSMN class. See Ferreira, Lachos and Bolfarine (2016) for details. For MSTT, MSSL2 and MSCN2, the generation is based on the multivariate scale mixtures of skew-normal (MSMSN) class. See Branco and Dey (2001) for details. For MSNC, the generation is based on the stochastic representation in Proposition 2.1 of Kahrari et al. (2016). For the MSTEC, MSSLEC and MSCEC models, the generation is based on the MSMSNC class. See Kahrari et al. (2017) for details.

Value

A matrix with the generated data.

Author(s)

Clecio Ferreira, Diego Gallardo and Camila Zeller.

References

Branco, M.D., Dey, D.K. (2001). A general class of multivariate skew-elliptical distributions. Journal of Multivariate Analysis 79, 99-113.

Ferreira, C.S., Lachos, V.H., Bolfarine, H. (2016). Likelihood-based inference for multivariate skew scale mixtures of normal distributions. AStA Advances in Statistical Analysis 100, 421-441.

Kahrari, F., Rezaei, M., Yousefzadeh, F., Arellano-Valle, R.B. (2016). On the multivariate skew-normal-Cauchy distribution. Statistics and Probability Letters, 117, 80-88.

Kahrari, F., Arellano-Valle, R.B., Rezaei, M., Yousefzadeh, F. (2017). Scale mixtures of skew-normal-Cauchy distributions. Statistics and Probability Letters, 126, 1-6.

Lange, K., Sinsheimer, J.S. (1993). Normal/independent distributions and their applications in robust regression. Journal of Computational and Graphical Statistics 2, 175-198.

Examples

```
rMSN(10, mu=c(0,0), Sigma=diag(2), lambda=c(1,-1)) ##bivariate MSN model
rMSNC(10, mu=0, Sigma=2, lambda=1) ##univariate MSNC model
rMSNC(10, mu=1:3, Sigma=2*diag(3), lambda=c(1,-1,0)) ##trivariate MSN model
```

---

solve2 *Computes the inverse of a matrix*

Description

Computes the inverse of a matrix using the LU decomposition.

Usage

```
solve2(A)
```

Arguments

A	an invertible square matrix.

Details

Use the LU decomposition to compute the inverse of a matrix. In some cases, solve produces error to invert a matrix whereas this decomposition provide a valid solution.

Value

A square matrix corresponding to the inverse of A.

Author(s)

Clecio Ferreira, Diego Gallardo and Camila Zeller

References

Bellman, R. (1987). Matrix Analysis, Second edition, Classics in Applied Mathematics, Society for Industrial and Applied Mathematics.

Horn, R. A. and C. R. Johnson (1985). Matrix Analysis, Cambridge University Press.

Examples

```
A=matrix(c(1,2,5,6),ncol=2)
solve2(A)
```

---

summary.skewMLRM *Print a summary for a object estimate.xxx*

Description

Summarizes the results for a object of the class "skewMLRM".

Usage

```
## S3 method for class 'skewMLRM'
summary(object, ...)
```

Arguments

`object`	an object of the class "skewMLRM". See details for supported models.
`...`	for extra arguments

Details

Supported models are:

In MSMN class: multivariate normal (MN), multivariate Student t (MT), multivariate slash (MSL), multivariate contaminated normal (MCN). See Lange and Sinsheimer (1993) for details.

In MSMSN class: multivariate skew-normal (MSN), multivariate skew-T (MSTT), multivariate skew-slash (MSSL2), multivariate skew-contaminated normal (MSCN2). See Zeller, Lachos and Vilca-Labra (2011) for details.

In MSSMN class: MSN, multivariate skew-t-normal (MSTN), multivariate skew-slash normal (MSSL), multivariate skew-contaminated normal (MSCN). See Louredo, Zeller and Ferreira (2021) for details.

In MSMSNC class: multivariate skew-normal-Cauchy (MSNC), multivariate skew-t-Expected-Cauchy (MSTEC), multivariate skew-slash-Expected-Cauchy (MSSLEC), multivariate skew-contaminated-Expected-Cauchy (MSCEC). See Kahrari et al. (2020) for details.

Note: the MSN distribution belongs to both, MSMSN and MSSMN classes.

The functions which generate an object of the class "skewMLRM" are

estimate.xxx: where xxx can be MN, MT, MSL, MCN, MSN, MSTN, MSSL, MSCN, MSTT, MSSL2, MSCN2, MSNC, MSTEC, MSSLEC or MSCEC.

choose.yyy: where yyy can be MSMN, MSSMN, MSMSN, MSMSNC or models.

choose2, mbackcrit, mbacksign and distMahal.

Value

A complete summary for the coefficients extracted from a skewMLRM object. If the object was generated by function distMahal, the summary is related to the Mahalanobis distances.

Author(s)

Clecio Ferreira, Diego Gallardo and Camila Zeller

References

Kahrari, F., Arellano-Valle, R.B., Ferreira, C.S., Gallardo, D.I. (2020) Some Simulation/computation in multivariate linear models of scale mixtures of skew-normal-Cauchy distributions. Communications in Statistics - Simulation and Computation. In press. DOI: 10.1080/03610918.2020.1804582

Lange, K., Sinsheimer, J.S. (1993). Normal/independent distributions and their applications in robust regression. Journal of Computational and Graphical Statistics 2, 175-198.

Louredo, G.M.S., Zeller, C.B., Ferreira, C.S. (2021). Estimation and influence diagnostics for the multivariate linear regression models with skew scale mixtures of normal distributions. Sankhya B. In press. DOI: 10.1007/s13571-021-00257-y

Zeller, C.B., Lachos, V.H., Vilca-Labra, F.E. (2011). Local influence analysis for regression models with scale mixtures of skew-normal distributions. Journal of Applied Statistics 38, 343-368.

Examples

```
data(ais, package="sn") ##Australian Institute of Sport data set
attach(ais)
##It is considered a bivariate regression model
##with Hg and SSF as response variables and
##Hc, Fe, Bfat and LBM as covariates
y<-cbind(Hg,SSF)
n<-nrow(y); m<-ncol(y)
X.aux=model.matrix(~Hc+Fe+Bfat+LBM)
p<-ncol(X.aux)
X<-array(0,dim=c(2*p,m,n))
for(i in 1:n) {
    X[1:p,1,i]=X.aux[i,,drop=FALSE]
    X[p+1:p,2,i]=X.aux[i,,drop=FALSE]
}
##See the covariate matrix X
##X

fit.MN=estimate.MN(y, X)     #fit the MN distribution
summary(fit.MN)              #summary for the fit
#
fit.MSN=estimate.MSN(y, X)   #fit the MSN distribution
summary(fit.MSN)             #summary for the fit
```

---

tgamma *Truncated gamma distribution*

Description

Compute the probability density and quantile functions for the truncated gamma distribution with shape and scale parameters, restricted to the interval (a,b).

Usage

```
dtgamma(x, shape, scale = 1, a = 0, b = Inf)
qtgamma(p, shape, scale = 1, a = 0, b = Inf)
```

Arguments

x	vector of quantiles
p	vector of probabilities
shape	shape parameter
scale	scale parameter
a	lower limit of range
b	upper limit of range

Value

dtgamma gives the density function for the truncated gamma distribution. qtgamma gives the quantile function for the truncated gamma distribution.

Note

Auxiliary function to compute the E step for the Slash and Skew-slash models.

Author(s)

Clecio Ferreira, Diego Gallardo and Camila Zeller

Examples

```
##probability density and quantile function of the truncated gamma
##model with shape and scale parameters equal to 1
##evaluated in 2 and 0.75, respectively
dtgamma(2, shape=1, a=1)
qtgamma(0.75, shape=1, a=1)
##standard gamma distribution with shape parameter 2 evaluated in 1
dtgamma(1, shape=2)
dgamma(1, shape=2)
```

---

vcov.skewMLRM *Calculate Variance-Covariance Matrix for a Fitted Model Object*

Description

Returns the variance-covariance matrix of the parameters of a fitted model object of the class "skewMLRM".

Usage

```
## S3 method for class 'skewMLRM'
vcov(object, ...)
```

Arguments

object	an object of the class "skewMLRM". See details for supported models.
...	for extra arguments

Details

Supported models are:

In MSMN class: multivariate normal (MN), multivariate Student t (MT), multivariate slash (MSL), multivariate contaminated normal (MCN). See Lange and Sinsheimer (1993) for details.

In MSMSN class: multivariate skew-normal (MSN), multivariate skew-T (MSTT), multivariate skew-slash (MSSL2), multivariate skew-contaminated normal (MSCN2). See Zeller, Lachos and Vilca-Labra (2011) for details.

In MSSMN class: MSN, multivariate skew-t-normal (MSTN), multivariate skew-slash normal (MSSL), multivariate skew-contaminated normal (MSCN). See Louredo, Zeller and Ferreira (2021) for details.

In MSMSNC class: multivariate skew-normal-Cauchy (MSNC), multivariate skew-t-Expected-Cauchy (MSTEC), multivariate skew-slash-Expected-Cauchy (MSSLEC), multivariate skew-contaminated-Expected-Cauchy (MSCEC). See Kahrari et al. (2020) for details.

Note: the MSN distribution belongs to both, MSMSN and MSSMN classes.

The functions which generate an object of the class "skewMLRM" compatible with vcov are

estimate.xxx: where xxx can be MN, MT, MSL, MCN, MSN, MSTN, MSSL, MSCN, MSTT, MSSL2, MSCN2, MSNC, MSTEC, MSSLEC or MSCEC.

choose.yyy: where yyy can be MSMN, MSSMN, MSMSN, MSMSNC or models.

choose2, mbackcrit and mbacksign.

Value

A matrix of the estimated covariances between the parameter estimates in the linear or non-linear predictor of the model. This should have row and column names corresponding to the parameter names given by the coef method.

Author(s)

Clecio Ferreira, Diego Gallardo and Camila Zeller

References

Kahrari, F., Arellano-Valle, R.B., Ferreira, C.S., Gallardo, D.I. (2020) Some Simulation/computation in multivariate linear models of scale mixtures of skew-normal-Cauchy distributions. Communications in Statistics - Simulation and Computation. In press. DOI: 10.1080/03610918.2020.1804582

Lange, K., Sinsheimer, J.S. (1993). Normal/independent distributions and their applications in robust regression. Journal of Computational and Graphical Statistics 2, 175-198.

Louredo, G.M.S., Zeller, C.B., Ferreira, C.S. (2021). Estimation and influence diagnostics for the multivariate linear regression models with skew scale mixtures of normal distributions. Sankhya B. In press. DOI: 10.1007/s13571-021-00257-y

Zeller, C.B., Lachos, V.H., Vilca-Labra, F.E. (2011). Local influence analysis for regression models with scale mixtures of skew-normal distributions. Journal of Applied Statistics 38, 343-368.

Examples

```
data(ais, package="sn") ##Australian Institute of Sport data set
attach(ais)
##It is considered a bivariate regression model
##with Hg and SSF as response variables and
##Hc, Fe, Bfat and LBM as covariates
y<-cbind(Hg,SSF)
n<-nrow(y); m<-ncol(y)
X.aux=model.matrix(~Hc+Fe+Bfat+LBM)
p<-ncol(X.aux)
X<-array(0,dim=c(2*p,m,n))
for(i in 1:n) {
    X[1:p,1,i]=X.aux[i,,drop=FALSE]
    X[p+1:p,2,i]=X.aux[i,,drop=FALSE]
}
##See the covariate matrix X
##X

fit.MN=estimate.MN(y, X)     #fit the MN distribution
vcov(fit.MN)                 #variance-covariance matrix
fit.MSN=estimate.MSN(y, X)   #fit the MSN distribution
vcov(fit.MSN)                #variance-covariance matrix
```

---

vech *Vectorize a symmetric matrix*

Description

vech takes the upper diagonal from a symmetric matrix and vectorizes it.

Usage

```
vech(x)
```

Arguments

`x`	a symmetric matrix.

Value

A vector with the components of the upper diagonal from the matrix, listed by row.

Note

For internal use.

Author(s)

Clecio Ferreira, Diego Gallardo and Camila Zeller.

Examples

```
A<-matrix(c(1,2,2,5),nrow=2)
##vectorized A matrix
B<-vech(A)
B
##reconstitute matrix A using B
xpnd(B,2)
```

`xpnd` *Reconstitute a symmetric matrix from a vector.*

Description

xpnd reconstitutes a symmetric matrix from a vector obtained with the vech function.

Usage

```
xpnd(x, nrow = NULL)
```

Arguments

`x`	vector with the components of the upper diagonal of the matrix
`nrow`	dimension of the matrix to be reconstitute.

Value

A symmetric matrix.

Note

For internal use.

Author(s)

Clecio Ferreira, Diego Gallardo and Camila Zeller.

Examples

```
A<-matrix(c(1,2,2,5),nrow=2)
##vectorized A matrix
B<-vech(A)
B
##reconstitute matrix A using B
xpnd(B,2)
```

Index