# Package 'proporz'

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Type Package

Title Proportional Apportionment

Version 1.5.0

**Description** Calculate seat apportionment for legislative bodies with various methods. The algorithms include divisor or highest averages methods

(e.g. Jefferson, Webster or Adams), largest remainder methods and biproportional apportionment.
Gaffke, N. & Pukelsheim, F. (2008) <doi:10.1016/j.mathsocsci.2008.01.004>

Oelbermann, K. F. (2016) <doi:10.1016/j.mathsocsci.2016.02.003>.

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Author Flavio Poletti [aut, cre, cph]

Maintainer Flavio Poletti <flavio.poletti@hotmail.ch>

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biproporz

Biproportional apportionment

# Description

Method to proportionally allocate seats among parties (or lists) and districts (or entities, regions), thus bi-proportional.

## Usage

```
biproporz(
   votes_matrix,
   district_seats,
   quorum,
   use_list_votes = TRUE,
   method = "round"
)
```

# Arguments

district\_seats Vector defining the number of seats per district. Must be the same length as ncol(votes\_matrix). Values are name-matched to votes\_matrix if both are named. If the number of seats per district should be assigned according to the number of votes (not the general use case), a single number for the total number of seats can be used.

# biproporz

quorum	Optional list of functions which take the votes_matrix and return a logical vector that denotes for each list/party whether they reached the quorum (i.e. are eligible for seats). The easiest way to do this is via quorum_any() or quorum_all(), see examples. Alternatively you can pass a precalculated logical vector. No quorum is applied if parameter is missing or NULL.
use_list_votes	By default (TRUE) it's assumed that each voter in a district has as many votes as there are seats in a district. Thus, votes are weighted according to the num- ber of available district seats with weight_list_votes(). Set to FALSE if votes_matrix shows the number of voters (e.g. they can only cast one vote for one party).
method	Defines which method is used to assign seats. The following methods are rec- ommended:
	<ul> <li>round: Uses the Sainte-Laguë/Webster method (rounding half up) for the upper and lower apportionment which is the standard for biproportional apportionment and the only method guaranteed to terminate.</li> </ul>
	• wto: "winner take one" works like "round" with a condition that the party that got the most votes in a district must get <i>at least</i> one seat ('Majorzbedingung') in said district. Seats in the upper apportionment are assigned with Sainte-Laguë/Webster. votes_matrix must have row and column names to use this method. See lower_apportionment() for more details.
	It is also possible to use any divisor method name listed in proporz(). If you want to use a different method for the upper and lower apportionment, provide a list with two entries.

# Details

Each party nominates a candidate list for every district. The voters vote for the parties of their district. The seat allocation is calculated in two steps:

- 1. In the so called upper apportionment the number of seats for each party (over all districts) is determined. Normally, the number of seats for each region are defined before the election and are independent of the vote counts.
- 2. In the so called lower apportionment the seats are distributed to the regional party list respecting the results from the upper apportionment.

Parties failing to reach quorums cannot get seats. This function does not handle seat assignment to candidates.

#### Value

Matrix with the same dimension as votes\_matrix containing the number of seats with the row and column divisors stored in attributes (hidden from print, see get\_divisors()).

# Note

The iterative process in the lower apportionment is only guaranteed to terminate with the default Sainte-Laguë/Webster method.

## References

Gaffke, Norbert; Pukelsheim, Friedrich (2008): Divisor methods for proportional representation systems: An optimization approach to vector and matrix apportionment problems. Mathematical Social Sciences, 56 (2), 166-184.

#### See Also

pukelsheim() for biproportional apportionment with data.frames as inputs.

### Examples

ceil\_at

Rounding with predefined thresholds

#### Description

Round x up to ceiling(x) if x-floor(x) >= threshold, otherwise round down to floor(x).

# Usage

ceil\_at(x, threshold)

## Arguments

Х	numeric vector or matrix $\geq 0$ (NaN is not supported)
threshold	threshold in [0,1] or "harmonic"/"geometric" to use harmonic or geometric mean
	thresholds

# Value

the rounded vector or matrix

# Examples

```
ceil_at(c(0.5, 1.5, 2.49, 2.5, 2.51), 0.5)
# compare to
round(c(0.5, 1.5, 2.49, 2.5, 2.51))
ceil_at(c(1.45, 2.45, 3.45), 0) # like floor()
ceil_at(c(1.45, 2.45, 3.45, 0.2), "geometric")
```

### Description

Functions to directly apply divisor apportionment methods instead of calling proporz() with a method parameter.

### Usage

```
divisor_round(votes, n_seats, quorum = 0)
divisor_floor(votes, n_seats, quorum = 0)
divisor_harmonic(votes, n_seats, quorum = 0)
divisor_geometric(votes, n_seats, quorum = 0)
divisor_ceiling(votes, n_seats, quorum = 0)
```

# Arguments

votes	numeric vector with number of votes for each party
n_seats	total number of seats
quorum	Vote threshold a party must reach. Used as quota of total votes within a district if less than 1 otherwise as number of votes.

# Details

Divisor methods are known under different names:

- d'hondt, jefferson, hagenbach-bischoff: use divisor\_floor()
- sainte-lague, webster: use divisor\_round()
- adams: use divisor\_ceiling()
- dean: use divisor\_harmonic()
- huntington-hill, hill-huntington: use divisor\_geometric()

All divisor functions call highest\_averages\_method() with a different sequence of divisors.

#### Value

The number of seats per party as a vector

# See Also

proporz()

#### Examples

```
votes = c("Party A" = 690, "Party B" = 400,
               "Party C" = 250, "Party D" = 120)
divisor_round(votes, 10)
divisor_floor(votes, 10)
divisor_ceiling(votes, 10)
divisor_ceiling(votes, 5)
divisor_geometric(votes, 10, quorum = 0.05)
divisor_harmonic(votes, 10)
```

finland2019

Finnish Parliamentary Elections Data (2019)

#### Description

Example data from the 2019 Finnish parliamentary elections. The data has been cleaned up and only contains information relevant for this package.

# Usage

finland2019

# Format

List containing two data.frames:

- votes\_df containing the number of votes for each party and district. 229 rows, 3 columns (party\_name, district\_name, votes)
- district\_seats\_df with the number of seats per district. 12 rows, 2 columns (district\_name, seats)

#### Source

https://tulospalvelu.vaalit.fi/EKV-2019/en/ladattavat\_tiedostot.html

# Examples

finland2019\$district\_seats\_df

head(finland2019\$votes\_df)

get\_divisors

# Description

Show the district and party divisors used to assign seats. This method provides easier access to divisors stored in attributes(...)\$divisors

# Usage

```
get_divisors(biproporz_result)
```

#### Arguments

biproporz\_result

a matrix created by biproporz() or a data.frame created by pukelsheim()

# Value

The district and party divisors in a list, each as a vector

# Examples

```
seats_matrix = biproporz(uri2020$votes_matrix, uri2020$seats_vector)
get_divisors(seats_matrix)
```

highest\_averages\_method

Highest averages method

# Description

Allocate seats proportionally for divisor methods.

## Usage

```
highest_averages_method(votes, n_seats, divisors)
```

# Arguments

votes	numeric vector with number of votes for each party
n_seats	total number of seats
divisors	sequence of divisors (length equal to the number of seats). If it is a single number (e.g. 0.5), a sequence is generated starting with it.

# Details

The highest averages method requires the number of votes for each party to be divided successively by a series of divisors. This produces a table of quotients, or averages, with a row for each divisor and a column for each party. The nth seat is allocated to the party whose column contains the nth largest entry in this table, up to the total number of seats available. (Wikipedia)

## Value

The number of seats per party as a vector

#### Examples

largest\_remainder\_method

Largest remainder method

# Description

Allocate seats based on the largest fractional remainder. The largest remainder method is also known as: Hamilton, Hare-Niemeyer or Vinton method.

#### Usage

```
largest_remainder_method(votes, n_seats, quorum = 0)
```

#### Arguments

votes	numeric vector with number of votes for each party
n_seats	total number of seats
quorum	Vote threshold a party must reach. Used as quota of total votes within a district if less than 1 otherwise as number of votes.

#### Details

The numbers of votes for each party is divided by a quota representing the number of votes required for a seat. Then, each party receives the rounded down quota value as seats. The remaining seats are given to the party with the largest remainder until all seats have been distributed.

#### Value

The number of seats per party as a vector

# lower\_apportionment

# Note

Only the quota total votes / total seats (which is used by the aforementioned methods) is implemented.

# See Also

proporz()

# Examples

```
votes = c(47000, 16000, 15800, 12000, 6100, 3100)
largest_remainder_method(votes, 10)
```

lower\_apportionment Calculate lower apportionment

# Description

Iterate and change column and row divisors such that the row and column sums of the seats matrix satisfies the constraints given by the upper apportionment.

#### Usage

lower\_apportionment(votes\_matrix, seats\_cols, seats\_rows, method = "round")

# Arguments

votes_matrix	matrix with votes by party in rows and votes by district in columns.
seats_cols	number of seats per column (districts/regions), predetermined or calculated with upper_apportionment().
seats_rows	number of seats per row (parties/lists), calculated with upper_apportionment().
method	Apportion method that defines how seats are assigned. The following methods are supported:
	• round: The default Sainte-Laguë/Webster method is the standard for bipro- portional apportionment and the only method guaranteed to terminate.
	• wto: "winner take one" works like "round" with a condition that the party that got the most votes in a district must get <i>at least</i> one seat ('Majorzbe-dingung'). The condition does not apply in a district if two or more parties have the same number of votes and there are not enough seats for these parties. A warning is issued in this case. Modify the votes matrix to explicitly break ties.
	• You can provide a custom function that rounds a matrix (i.e. the the votes_matrix divided by party and list divisors).
	• It is possible to use any divisor method name listed in propert()

• It is possible to use any divisor method name listed in proporz().

#### Details

The result is obtained by an iterative process ('Alternate Scaling Algorithm', see Reference). Initially, for each district a divisor is chosen using the highest averages method for the votes allocated to each regional party list in this region. For each party a party divisor is initialized with 1.

Effectively, the objective of the iterative process is to modify the regional divisors and party divisors so that the number of seats in each regional party list equals the number of their votes divided by both the regional and the party divisors.

The following two correction steps are executed until this objective is satisfied:

- modify the party divisors such that the apportionment within each party is correct with the chosen rounding method,
- modify the regional divisors such that the apportionment within the region is correct with the chosen rounding method.

# Value

A seat matrix with district (columns) and party (rows) divisors stored in attributes.

#### References

Oelbermann, K. F. (2016): Alternate scaling algorithm for biproportional divisor methods. Mathematical Social Sciences, 80, 25-32.

## See Also

biproporz(), upper\_apportionment()

## Examples

```
votes_matrix = matrix(c(123,912,312,45,714,255,815,414,215), nrow = 3)
district_seats = c(7,5,8)
party_seats = c(5,11,4)
```

lower\_apportionment(votes\_matrix, district\_seats, party\_seats)

```
lower_apportionment(vm, ua$district, ua$party, method = "wto")
```

```
# compare to standard method
lower_apportionment(vm, ua$district, ua$party, method = "round")
```

pivot\_to\_matrix Pivot long data.frame to wide matrix and vice versa

#### Description

Create a matrix in 'wide' format from a data.frame with 3 columns with pivot\_to\_matrix or create a data.frame in long format from a matrix with pivot\_to\_df.

#### Usage

```
pivot_to_matrix(df_long)
```

pivot\_to\_df(matrix\_wide, value\_colname = "values")

#### Arguments

df_long	data.frame in long format with exactly 3 columns
matrix_wide	matrix in wide format
value_colname	name for the new value column in the resulting data.frame

# Details

These pivot functions are used to prepare data for biproporz() in pukelsheim(). They are not supposed to cover general use cases or provide customization. They mainly exist because reshape is hard to handle and the package should have no dependencies.

#### Value

A data.frame with 3 columns or a matrix. Note that the results are sorted by the first and second column (data.frame) or row/column names (matrix).

# Examples

pivot\_to\_df(pivot\_to\_matrix(df)) == df[order(df[[1]], df[[2]]),]

proporz

# Proportional apportionment

# Description

Calculate seat apportionment for legislative bodies.

#### Usage

proporz(votes, n\_seats, method, quorum = 0)

### Arguments

votes	numeric vector with number of votes for each party
n_seats	total number of seats
method	Apportionment method to use, as character. Not case sensitive. See details.
quorum	Vote threshold a party must reach. Used as quota of total votes within a district if less than 1 otherwise as number of votes.

#### Details

The following methods are available:

- d'hondt, jefferson, hagenbach-bischoff, floor: use divisor\_floor()
- sainte-lague, webster, round: use divisor\_round()
- adams, ceiling: use divisor\_ceiling()
- dean, harmonic: use divisor\_harmonic()
- huntington-hill, hill-huntington, geometric: use divisor\_geometric()
- hare-niemeyer, hamilton, vinton, largest\_remainder\_method: use largest\_remainder\_method()

# Value

The number of seats per party as a vector

#### Note

Seats can also be apportioned among regions instead of parties. The parameter votes is then normally used with census data (e.g. population counts).

## pukelsheim

# Examples

```
votes = c("Party A" = 651, "Party B" = 349, "Party C" = 50)
proporz(votes, 10, "sainte-lague")
proporz(votes, 10, "hill-huntington")
proporz(votes, 10, "hill-huntington", quorum = 0.05)
proporz(votes, 10, "jefferson", quorum = 70)
```

pukelsheim Biproportional apportionment with data frames

#### Description

Method to proportionally allocate seats among parties/lists and districts/regions/entities ('Doppelter Pukelsheim').

#### Usage

```
pukelsheim(
  votes_df,
  district_seats_df,
  quorum,
  new_seats_col = "seats",
  use_list_votes = TRUE,
  winner_take_one = FALSE
)
```

#### Arguments

```
votes_df data.frame (long format) with 3 columns (actual colnames can differ):
```

- party id/name
- district id/name

```
• votes district_seats_df
```

data.frame with 2 columns (actual colnames can differ):

- district id/name
- number of seats for a district

```
quorum Optional list of functions which take the votes_matrix and return a logical vector that denotes for each list/party whether they reached the quorum (i.e. are eligible for seats). The easiest way to do this is via quorum_any() or quorum_all(), see examples. Alternatively you can pass a precalculated logical vector. No quorum is applied if parameter is missing or NULL.
```

new_seats_col	name of the new column
use_list_votes	By default (TRUE) it's assumed that each voter in a district has as many votes
	as there are seats in a district. Set to ${\tt FALSE}$ if <code>votes_df</code> shows the number of
	voters (e.g. they can only vote for one party).
winner_take_one	
	Set to TRUE if the party that got the most votes in a district must get at least one
	seat ('Majorzbedingung') in this district. Default is FALSE.

#### Details

Each party nominates a candidate list for every district. The voters vote for the parties of their district. The seat allocation is calculated in two steps:

- 1. In the so called upper apportionment the number of seats for each party (over all districts) is determined.
- 2. In the so called lower apportionment the seats are distributed to the regional party list respecting the results from the upper apportionment.

Parties failing to reach quorums cannot get seats. This function does not handle seat assignment to candidates.

If you want to use other apportion methods than Sainte-Laguë use biproporz().

#### Value

A data.frame like votes\_df with a new column denoting the number seats per party and district. Party and district divisors stored in attributes in attributes (hidden from print, see get\_divisors()).

# See Also

This function calls **biproporz()** after preparing the input data.

# Examples

quorum\_functions Create quorum functions for biproportional apportionment

#### Description

quorum\_any() and quorum\_all() are used for the quorum parameter in biproporz() or pukelsheim() and help describe how quorums should be applied previous to seat distributions.

# Usage

```
quorum_all(any_district, total)
```

quorum\_any(any\_district, total)

#### Arguments

any_district	Vote threshold a party must reach in <i>at least</i> one district. Used as share of total votes within a district if less than 1 otherwise as number of votes. Must be greater than 0. Uses reached_quorum_any_district().
total	Vote threshold a party must reach for all votes cast. Used as share of total votes if less than 1, otherwise as number of votes. Must be greater than 0. Uses reached_quorum_total().

## Details

There's a difference in how the functions work. With quroum\_any, *at least one* quorum must be reached. With quorum\_all *all* (i.e. both) quorums must be reached. If you only use one parameter, quorum\_any() and quorum\_all() are identical.

#### Value

a function which, when called with function(votes\_matrix), returns a boolean vector with length equal to the number of lists/parties (votes\_matrix rows). The vector shows whether a party has reached any/all quorums.

# Examples

```
votes_matrix = matrix(c(502, 55, 80, 10, 104, 55, 0, 1), ncol = 2)
dimnames(votes_matrix) <- list(c("A", "B", "C", "D"), c("Z1", "Z2"))
seats = c(Z1 = 50, Z2 = 20)
# use as parameter in biproporz or pukelsheim (general use case)
biproporz(votes_matrix, seats, quorum = quorum_any(any_district = 0.1, total = 100))
biproporz(votes_matrix, seats, quorum = quorum_all(any_district = 0.1, total = 100))
biproporz(votes_matrix, seats, quorum = quorum_any(any_district = 0.1, total = 100))
```

```
biproporz(votes_matrix, seats, quorum = quorum_any(total = 100))
biproporz(votes_matrix, seats, quorum = quorum_any(total = 0.5))
# the quorum parameter also accepts vectors (e.g. calculated elsewhere)
biproporz(votes_matrix, seats, quorum = c(FALSE, TRUE, TRUE, TRUE))
```

reached\_quorum\_any\_district

Check if lists/parties have reached a quorum in at least one district

## Description

Base implementation, used by quorum\_any() and quorum\_all().

# Usage

reached\_quorum\_any\_district(votes\_matrix, quorum\_districts)

### Arguments

votes\_matrix votes matrix quorum\_districts

Vote threshold a party must reach in *at least* one district. Used as quota of total votes within a district if less than 1 otherwise as number of votes. Must be greater than 0.

# Value

boolean vector with length equal to the number of lists/parties (votes\_matrix rows) whether they reached the quorum or not

# See Also

reached\_quorum\_total()

# Examples

```
(vm = matrix(c(239, 10, 308, 398, 20, 925), nrow = 3))
reached_quorum_any_district(vm, 25)
```

reached\_quorum\_total Check if lists/parties have reached the quorum for all votes

# Description

Base implementation, used by quorum\_any() and quorum\_all().

#### Usage

```
reached_quorum_total(votes_matrix, quorum_total)
```

# Arguments

votes_matrix	votes matrix
quorum_total	Vote threshold a party must reach for all votes cast. Used as quota of total votes
	if less than 1, otherwise as number of votes. Must be greater than 0.

# Value

boolean vector with length equal to the number of lists/parties (votes\_matrix rows) whether they reached the quorum or not

# See Also

reached\_quorum\_any\_district()

# Examples

(vm = matrix(c(239, 10, 308, 398, 20, 925), nrow = 3))
reached\_quorum\_total(vm, 35)

```
run_app
```

Use biproportional apportionment interactively in a shiny app

# Description

Use biproportional apportionment interactively in a shiny app

#### Usage

run\_app(votes\_matrix = NULL, district\_seats = NULL)

# Arguments

votes\_matrix optional votes\_matrix to load upon start district\_seats optional district\_seats to load upon start

# Value

Calling the function starts the shiny app

## Examples

```
if(interactive()){
    # You need to have the packages 'shiny' and 'shinyMatrix' installed to run the app
    run_app()
   # It's possible to load a matrix with the app
    run_app(uri2020$votes_matrix, uri2020$seats_vector)
}
```

Calculate upper apportionment upper\_apportionment

# Description

In the upper apportionment, the seats for each party are computed with a highest averages method. This determines how many of all seats each party deserves due to the total of all their votes (that is the sum of the votes for all regional lists of that party). Analogical, the same highest averages method is used to determine how many of all seats each region deserves.

# Usage

```
upper_apportionment(
  votes_matrix,
  district_seats,
  use_list_votes = TRUE,
 method = "round"
)
```

#### Arguments

votes_matrix	Vote count matrix with votes by party in rows and votes by district in columns
district_seats	Vector defining the number of seats per district. Must be the same length as ncol(votes_matrix). Values are name-matched to votes_matrix if both are named. If the number of seats per district should be assigned according to the number of votes (not the general use case), a single number for the total number of seats can be used.
use_list_votes	By default (TRUE) it's assumed that each voter in a district has as many votes as there are seats in a district. Thus, votes are weighted according to the num- ber of available district seats with weight_list_votes(). Set to FALSE if votes_matrix shows the number of voters (e.g. they can only cast one vote for one party).
method	Apportion method that defines how seats are assigned, see proporz(). Default is the Saintë-Lague/Webster method.

## uri2020

# Value

A named list with district seats (for votes\_matrix columns) and party seats (for rows).

#### Note

The results from the upper apportionment are final results for the number of the seats of one party (and analogically for the number of the seats of one region) within the whole voting area, the lower apportionment will only determine where (which regions) the party seats are allocated. Thus, after the upper apportionment is done, the final strength of a party/region within the parliament is definite.

# See Also

biproporz(), lower\_apportionment()

#### Examples

```
votes_matrix = matrix(c(123,912,312,45,714,255,815,414,215), nrow = 3)
district_seats = c(7,5,8)
```

upper\_apportionment(votes\_matrix, district\_seats)

uri2020

Election Data for the Cantonal Council of Uri (2020)

## Description

Example election data from the 2020 election for the cantonal council of Uri (Landrat) in Switzerland. The data has been extracted from the report "Landratswahlen 2020: Statistische Auswertung".

#### Usage

uri2020

#### Format

List containing:

- votes\_matrix the number of votes for each party and district (4 rows, 4 columns)
- seats\_vector with the number of seats per district (length 4)

# Source

https://www.ur.ch/abstimmungen/termine/9322

weight\_list\_votes Create weighted votes matrix

### Description

Weigh list votes by dividing the votes matrix entries by the number of seats per district. This method is used in upper\_apportionment() if use\_list\_votes is TRUE (default). The weighted votes are not rounded.

# Usage

weight\_list\_votes(votes\_matrix, seats\_district)

# Arguments

votes\_matrix votes matrix
seats\_district seats per district, vector with same length as ncol(votes\_matrix))

# Value

the weighted votes\_matrix

# Examples

weight\_list\_votes(uri2020\$votes\_matrix, uri2020\$seats\_vector)

zug2018

Election Data for the Cantonal Council of Zug (2018)

## Description

Example election data from the 2018 election for the cantonal council of Zug (Kantonsrat) in Switzerland.

# Usage

zug2018

# Format

An object of class data. frame with 267 rows and 49 columns.

## Source

Kanton Zug (01.07.2022, 10:27:58). Kantonsratswahl 2018 (CSV). https://wab.zug.ch/elections/kantonsratswahl-2018/data-csv

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