Package ‘hlrhotrix’

April 21, 2026

Title Algebraic Operations and Visualisation for Hl-Rhotrices

Version 0.1.0

Date 2026-04-17

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Description Provides constructors for hl-rhotrices of dimension 2, 4 and 6, together with computation of the determinant, adjoint, inverse and eigenvalues under the Robust Multiplication Method (RMM). A 'ggplot2'-based function visualises the rhomboidal layout and the decomposition into principal minors.

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RoxygenNote 7.3.1

Depends R (>= 4.0.0)

Imports ggplot2, ggforce, patchwork

Suggests knitr, rmarkdown

VignetteBuilder knitr

NeedsCompilation no

Repository CRAN

Date/Publication 2026-04-21 19:32:29 UTC

Contents

`adj_hl` *Adjoint of an hl-rhotrix*

Description

Computes the adjoint following Definitions 3.5 and 3.6. Each minor's adjoint is scaled by the product of determinants of all other minors. The sign of the M2 block follows Remark 3.4.

Usage

```
adj_hl(rho)
```

Arguments

`rho`	An object of class `hl_rhotrix`.

Value

An object of class `hl_rhotrix` representing $\mathrm{adj}(R)$.

Examples

```
A <- make_R4(a11=-2, a21=3, c11=4, a12=1,
             a31=1, c21=2, c12=-1, a13=0,
             a32=1, c22=1, a23=0, a33=3)
adj_hl(A)
```

char_poly_hl *Characteristic polynomial of a 2-dimensional hl-rhotrix*

Description

Returns the characteristic polynomial $\Delta(t) = t^2 + \mathrm{tr}(A)\,t + \det(A)$

Usage

```
char_poly_hl(rho)
```

Arguments

rho	An object of class `hl_rhotrix` with `dim = 2`.

Value

A named list with elements:

coefficients Named vector `c(constant, linear, quadratic)` for use with `polyroot`.

poly_string Human-readable string of the polynomial.

trace Trace of the rhotrix.

determinant Determinant of the rhotrix.

Examples

```
R <- make_R2(a11 = 5, a21 = 3, a12 = 6, a22 = 2)
char_poly_hl(R)
```

det_hl *Determinant of an hl-rhotrix*

Description

Computes the determinant under the Robust Multiplication Method (RMM)

$$\det(R_n) = \det\Big(\prod_k A_{2k}\Big) \otimes \det\Big(\prod_l M_{2l}\Big)$$

where the sign of the M2 block is -1 when there is exactly one M2 minor, and $+1$ otherwise.

Usage

```
det_hl(rho)
```

Arguments

rho	An object of class hl_rhotrix.

Value

A numeric scalar.

Examples

```
A <- make_R4(a11=-2, a21=3, c11=4, a12=1,
             a31=1, c21=2, c12=-1, a13=0,
             a32=1, c22=1, a23=0, a33=3)
det_hl(A)   # -36

R <- make_R2(a11=5, a21=3, a12=6, a22=2)
det_hl(R)   # -8
```

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eigenvalues_hl *Eigenvalues of a 2-dimensional hl-rhotrix*

Description

Computes the eigenvalues as roots of the characteristic polynomial $\Delta(t) = t^2 + \mathrm{tr}(A)\,t + \det(A) = 0$

Usage

```
eigenvalues_hl(rho)
```

Arguments

rho	An object of class hl_rhotrix with dim = 2.

Value

A numeric or complex vector of length 2.

Examples

```
R <- make_R2(a11 = 5, a21 = 3, a12 = 6, a22 = 2)
eigenvalues_hl(R)   # -8 and 1
```

eigenvalues_hl_high *Eigenvalues of a high-dimensional hl-rhotrix (per R2 minor)*

Description

Applies `eigenvalues_hl` to each principal R2 minor of a high hl-rhotrix. As noted in the manuscript's concluding remarks, the computation of eigenvalues for high hl-rhotrices using the basal R2 is an open research direction.

Usage

```
eigenvalues_hl_high(rho)
```

Arguments

`rho`	An object of class `hl_rhotrix` (any dimension).

Value

A list; each element is a list with `minor` (name) and `eigenvalues` (length-2 vector) for one R2 minor.

Examples

```
A <- make_R4(a11=-2, a21=3, c11=4, a12=1,
             a31=1, c21=2, c12=-1, a13=0,
             a32=1, c22=1, a23=0, a33=3)
eigenvalues_hl_high(A)
```

inv_hl *Inverse of an hl-rhotrix*

Description

Computes $R^{-1} = \frac{1}{\det(R)} \cdot \mathrm{adj}(R)$

Usage

```
inv_hl(rho)
```

Arguments

`rho`	An object of class `hl_rhotrix`.

Value

An object of class `hl_rhotrix` representing R^{-1}.

Examples

```
A <- make_R4(a11=-2, a21=3, c11=4, a12=1,
             a31=1, c21=2, c12=-1, a13=0,
             a32=1, c22=1, a23=0, a33=3)
inv_hl(A)
```

`make_R2` *Create a 2-dimensional hl-rhotrix*

Description

Constructs the basal hl-rhotrix of dimension 2:

$$\begin{pmatrix} & a_{11} & \\ a_{21} & & a_{12} \\ & a_{22} & \end{pmatrix}$$

Usage

```
make_R2(a11, a21, a12, a22)
```

Arguments

`a11`	Top entry (diagonal / vertical axis).
`a21`	Left entry (symmetric).
`a12`	Right entry (symmetric).
`a22`	Bottom entry (diagonal / vertical axis).

Value

An object of class `hl_rhotrix` with `dim = 2`.

References

Isere (2018). Even Dimensional Rhotrix. *Notes on Number Theory and Discrete Mathematics*, 24(2), 125-133.

Examples

```
R <- make_R2(a11 = 5, a21 = 3, a12 = 6, a22 = 2)
det_hl(R)       # -8
eigenvalues_hl(R)  # -8 and 1
```

make_R4 *Create a 4-dimensional hl-rhotrix*

Description

Constructs a 4-dimensional hl-rhotrix in the rhomboidal layout:

```
       a11
    a21  c11  a12
 a31  c21  c12  a13
    a32  c22  a23
       a33
```

The rhotrix is decomposed into two principal R2 minors (A21, A22) and one M2 minor matrix (M21) following Theorem 3.1.

Usage

```
make_R4(a11, a21, c11, a12, a31, c21, c12, a13, a32, c22, a23, a33)
```

Arguments

a11, a33	Outer diagonal entries.
a21, a12	First-layer symmetric entries.
c11, c22	Inner diagonal entries (second R2 minor).
a31, a13	Second-layer symmetric entries (first R2 minor).
c21, c12	Inner symmetric entries (second R2 minor).
a32, a23	Third-layer symmetric entries (M2 minor).

Value

An object of class hl_rhotrix with dim = 4.

Examples

```
A <- make_R4(
  a11 = -2,
  a21 =  3, c11 = 4,  a12 = 1,
  a31 =  1, c21 = 2,  c12 = -1, a13 = 0,
  a32 =  1, c22 = 1,  a23 = 0,
  a33 =  3
)
det_hl(A)   # -36
```

make_R6 *Create a 6-dimensional hl-rhotrix*

Description

Constructs a 6-dimensional hl-rhotrix in the rhomboidal layout (see Theorem 3.3 of the manuscript). Decomposed into three principal R2 minors (A21, A22, A23) and three M2 minor matrices (M21, M22, M23).

Usage

```
make_R6(
  a11,
  a21,
  c11,
  a12,
  a31,
  c21,
  a22,
  c12,
  a13,
  a41,
  c31,
  a32,
  a23,
  c13,
  a14,
  a42,
  c32,
  a33,
  c23,
  a24,
  a43,
  c33,
  a34,
  a44
)
```

Arguments

a11, a44	Outermost diagonal entries.
a21, a12, a41, a14	First and fourth layer symmetric entries.
c11, c33	Outer diagonal entries of the second R2 minor.
a31, a13, a42, a24	Second R2 minor symmetric entries.

c21, c12, c31, c13	Outer symmetric entries of the second R2 minor.
a22, a33	Inner diagonal entries of the third R2 minor.
a32, a23	Inner symmetric entries of the third R2 minor.
c32, c23	Inner symmetric entries of the third M2 minor.
a43, a34	Fifth-layer symmetric entries (second M2 minor).

Value

An object of class hl_rhotrix with dim = 6.

Examples

```
R6 <- make_R6(
  a11=1, a21=2, c11=3, a12=4,
  a31=5, c21=6, a22=7, c12=8, a13=9,
  a41=2, c31=3, a32=4, a23=5, c13=6, a14=7,
  a42=1, c32=2, a33=3, c23=4, a24=5,
  a43=6, c33=7, a34=8, a44=9
)
det_hl(R6)  # 15360
```

---

plot_rhombus_gg *Rhomboidal layout diagram of an hl-rhotrix*

Description

Produces a publication-quality ggplot diagram showing the rhomboidal structure of an hl-rhotrix of dimension 2, 4 or 6. Nodes and regions are colour-coded by minor membership:

- **Blue**: principal minor rhotrices A_{2k} (R2), entries on the vertical axis.
- **Amber**: minor matrices M_{2l} (M2), symmetric off-axis entries.

Usage

```
plot_rhombus_gg(
  dim,
  gap = 1,
  node_r = 0.28,
  show_labels = TRUE,
  show_regions = TRUE,
  show_minor_labels = TRUE,
  show_legend = TRUE,
  show_edges = TRUE,
  title = NULL,
  subtitle = NULL,
  base_size = 11
)
```

Arguments

dim	Integer. Dimension of the hl-rhotrix: 2, 4 or 6.
gap	Numeric. Spacing between node centres (default 1).
node_r	Numeric. Radius of each node circle (default 0.28).
show_labels	Logical. Display entry labels inside nodes using plotmath subscript notation (default TRUE).
show_regions	Logical. Draw dashed bounding rectangles around each minor (default TRUE).
show_minor_labels	Logical. Label each minor region with its name (default TRUE).
show_legend	Logical. Display the colour legend below the diagram (default TRUE).
show_edges	Logical. Draw connector lines between nodes of the same minor (default TRUE).
title	Character. Plot title. NULL for no title.
subtitle	Character. Plot subtitle. NULL for no subtitle.
base_size	Numeric. Base font size (default 11).

Details

The function returns a standard ggplot object. Use ggplot2::ggsave() to export to PDF, PNG, SVG or EPS.

Value

A ggplot object.

Examples

```
library(ggplot2)

# Dimension 2
plot_rhombus_gg(2)

# Dimension 4 with title
p <- plot_rhombus_gg(4,
  title    = "4-dimensional hl-rhotrix",
  subtitle = "R2 minors (blue) and M2 matrices (amber)")
print(p)

# Dimension 6
plot_rhombus_gg(6, gap = 1.1, node_r = 0.26)


# Three-panel figure using patchwork
library(patchwork)
plot_rhombus_gg(2) + plot_rhombus_gg(4) + plot_rhombus_gg(6)
```

print.hl_rhotrix *Print method for hl_rhotrix*

Description

Print method for hl_rhotrix

Usage

```
## S3 method for class 'hl_rhotrix'
print(x, digits = 4, ...)
```

Arguments

x	An object of class hl_rhotrix.
digits	Number of significant digits (default 4).
...	Ignored.

Value

Invisibly returns x.

summary_hl *Summary of an hl-rhotrix with all computed invariants*

Description

Prints the hl-rhotrix structure together with its determinant, adjoint, inverse and eigenvalues.

Usage

```
summary_hl(rho)
```

Arguments

rho	An object of class hl_rhotrix.

Value

Invisibly returns a list with det, adj, inv and eigenvalues.

Examples

```
A <- make_R4(a11=-2, a21=3, c11=4, a12=1,
             a31=1, c21=2, c12=-1, a13=0,
             a32=1, c22=1, a23=0, a33=3)
summary_hl(A)
```

`trace_hl` *Trace of a 2-dimensional hl-rhotrix*

Description

The trace is the sum of entries along the vertical axis (i.e., $a_{11} + a_{22}$).

Usage

```
trace_hl(rho)
```

Arguments

`rho`	An object of class `hl_rhotrix` with `dim = 2`.

Value

A numeric scalar.

Examples

```
R <- make_R2(a11 = 5, a21 = 3, a12 = 6, a22 = 2)
trace_hl(R)  # 7
```

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