## Package 'carbondate'

January 31, 2024

Title Calibration and Summarisation of Radiocarbon Dates

Version 1.0.1

Description Performs Bayesian non-parametric calibration of multiple related radiocarbon determinations, and summarises the calendar age information to plot their joint calendar age density (see Heaton (2022) <doi:10.1111/rssc.12599>).
 Also models the occurrence of radiocarbon samples as a variable-rate (inhomogeneous) Poisson process, plotting the posterior estimate for the occurrence rate of the samples over calen-

dar time, and providing information about potential change points.

License GPL (>= 3)

Encoding UTF-8

**Suggests** knitr, rmarkdown, testthat (>= 3.0.0)

Config/testthat/edition 3

**Depends** R (>= 3.5.0)

LazyData true

Imports graphics, grDevices, stats, utils

URL https://github.com/TJHeaton/carbondate,

https://tjheaton.github.io/carbondate/

BugReports https://github.com/TJHeaton/carbondate/issues

VignetteBuilder knitr

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Author Timothy J Heaton [aut, cre, cph]

(<https://orcid.org/0000-0002-9994-142X>),
Sara Al-assam [aut, cph] (<https://orcid.org/0000-0001-7280-6802>)

Maintainer Timothy J Heaton <T.Heaton@leeds.ac.uk>

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#### Description

58 radiocarbon determinations collated by Dale Guthrie, R. related to Alces (moose) in Yukon and Alaska. Samples are restricted to those between 25,000–6000 <sup>14</sup>C yrs BP.

## **Reference:**

Dale Guthrie, R. New carbon dates link climatic change with human colonization and Pleistocene extinctions. *Nature* **441**, 207–209 (2006). https://doi.org/10.1038/nature04604

#### Usage

alces

## Format

alces:

A data frame with 58 rows and 7 columns: **lab\_code** The sample code for the <sup>14</sup>C laboratory **site\_code** The site/museum code **location** The location of the sample **c14\_age** The observed <sup>14</sup>C ages of the samples (in <sup>14</sup>C yr BP) **c14\_sig** The uncertainty in the observed <sup>14</sup>C ages reported by the radiocarbon laboratory **f14c** The observed F<sup>14</sup>C concentrations **f14c\_sig** The uncertainty in the observed F<sup>14</sup>C concentrations reported by the radiocarbon laboratory **f14c** ages The uncertainty in the observed F<sup>14</sup>C concentrations reported by the radiocarbon laboratory **f14c\_sig** The uncertainty in the observed F<sup>14</sup>C concentrations reported by the radiocarbon laboratory **f14c\_sig** The uncertainty in the observed F<sup>14</sup>C concentrations reported by the radiocarbon laboratory **f14c** ages The uncertainty in the observed F<sup>14</sup>C concentrations reported by the radiocarbon laboratory **f14c** ages The uncertainty in the observed F<sup>14</sup>C concentrations reported by the radiocarbon laboratory **f14c** ages The uncertainty in the observed F<sup>14</sup>C concentrations reported by the radiocarbon laboratory **f14c** ages The uncertainty in the observed F<sup>14</sup>C concentrations reported by the radiocarbon laboratory **f14c** ages The uncertainty in the observed F<sup>14</sup>C concentrations reported by the radiocarbon laboratory **f14c** ages The uncertainty in the observed F<sup>14</sup>C concentrations reported by the radiocarbon laboratory **f14c** ages The uncertainty in the observed F<sup>14</sup>C concentrations reported by the radiocarbon laboratory

#### Source

https://doi.org/10.1038/nature04604

armit

Example real-life data - Population Decline in Iron Age Ireland

## Description

2021 radiocarbon determinations collated by Armit et al. from archaeological groups operating in Ireland, to investigate whether a wetter environment around 2700 cal yr BP led to a population collapse.

#### **Reference:**

Armit, I., Swindles, G.T., Becker, K., Plunkett, G., Blaauw, M., 2014. Rapid climate change did not cause population collapse at the end of the European Bronze Age. *Proceedings of the National Academy of Sciences* **111**, 17045–17049.

#### Usage

armit

## Format

armit:

A data frame with 2021 rows and 4 columns:

**c14\_age** The observed <sup>14</sup>C ages of the samples (in <sup>14</sup>C yr BP)

c14\_sig The uncertainty in the observed <sup>14</sup>C ages reported by the radiocarbon laboratory

**f14c** The observed  $F^{14}C$  concentrations

**f14c\_sig** The uncertainty in the observed F<sup>14</sup>C concentrations reported by the radiocarbon laboratory

#### Source

http://doi.org/10.1073/pnas.1408028111

bison

Example real-life data - Bison in Yukon and Alaska

#### Description

64 radiocarbon determinations collated by Dale Guthrie, R. related to Bison in Yukon and Alaska. Samples are restricted to those between 25,000-6000 <sup>14</sup>C yrs BP.

#### **Reference:**

Dale Guthrie, R. New carbon dates link climatic change with human colonization and Pleistocene extinctions. *Nature* **441**, 207–209 (2006). https://doi.org/10.1038/nature04604

## Usage

bison

## Format

## bison:

A data frame with 64 rows and 7 columns:

**lab\_code** The sample code for the <sup>14</sup>C laboratory

site\_code The site/museum code

location The location of the sample

**c14\_age** The observed <sup>14</sup>C ages of the samples (in <sup>14</sup>C yr BP)

c14\_sig The uncertainty in the observed <sup>14</sup>C ages reported by the radiocarbon laboratory

**f14c** The observed  $F^{14}C$  concentrations

**f14c\_sig** The uncertainty in the observed F<sup>14</sup>C concentrations reported by the radiocarbon laboratory

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## buchanan

#### Source

https://doi.org/10.1038/nature04604

buchanan

Example real-life data - Palaeo-Indian demography

## Description

628 radiocarbon determinations collated by Buchanan et al. representing the ages of distinct archaeological sites found across Canada and North America during the time of the palaeoindians.

#### **Reference:**

Buchanan, B., Collard, M., Edinborough, K., 2008. Paleoindian demography and the extraterrestrial impact hypothesis. *Proceedings of the National Academy of Sciences* **105**, 11651–11654.

#### Usage

buchanan

## Format

buchanan:

A data frame with 628 rows and 4 columns:

c14\_age The observed <sup>14</sup>C ages of the samples (in <sup>14</sup>C yr BP)

c14\_sig The uncertainty in the observed <sup>14</sup>C ages reported by the radiocarbon laboratory

**f14c** The observed  $F^{14}C$  concentrations

**f14c\_sig** The uncertainty in the observed F<sup>14</sup>C concentrations reported by the radiocarbon laboratory

#### Source

http://doi.org/10.1073/pnas.0803762105

CalibrateSingleDetermination

Calibrate a Single Radiocarbon Determination

#### Description

Uses the supplied calibration curve to calibrate a single radiocarbon determination and uncertainty (expressed either in terms of radiocarbon age, or as an  $F^{14}C$  concentration) and obtain its calendar age probability density estimate.

## Usage

```
CalibrateSingleDetermination(
  rc_determination,
  rc_sigma,
  calibration_curve,
  F14C_inputs = FALSE,
  resolution = 1,
  plot_output = FALSE,
  plot_cal_age_scale = "BP",
  interval_width = "2sigma",
  bespoke_probability = NA,
  denscale = 3,
  plot_pretty = TRUE
)
```

## Arguments

rc_determination		
	A single observed radiocarbon determination provided either as the radiocarbon age (in $^{14}$ C yr BP) or the F $^{14}$ C concentration.	
rc_sigma	The corresponding measurement uncertainty of the radiocarbon determination (must be in the same units as above, i.e., reported as ${}^{14}C$ age or $F^{14}C$ )	
calibration_cur	ve	
	A dataframe which must contain one column calendar_age_BP, and also columns c14_age and c14_sig or f14c and f14c_sig (or both sets). This format matches the curves supplied with this package, e.g., intcal20, intcal13, which contain all 5 columns.	
F14C_inputs	TRUE if the provided rc_determination is an $F^{14}C$ concentration and FALSE if it is a radiocarbon age. Defaults to FALSE.	
resolution	The distance between the calendar ages at which to calculate the calendar age probability. Default is 1.	
plot_output	TRUE if you wish to plot the determination, the calibration curve, and the posterior calibrated age estimate on the same plot. Defaults to FALSE	
plot_cal_age_scale		
	Only for usage when plot_output = TRUE. The calendar scale to use for the x-axis. Allowed values are "BP", "AD" and "BC". The default is "BP", corresponding to plotting in cal yr BP.	
interval_width	Only for usage when plot_output = TRUE. The confidence intervals to show for the calibration curve and for the highest posterior density ranges. Choose from one of "1sigma" (68.3%), "2sigma" (95.4%) and "bespoke". Default is "2sigma".	
bespoke_probability		
	The probability to use for the confidence interval if "bespoke" is chosen above. E.g. if 0.95 is chosen, then the 95% confidence interval is calculated. Ignored if "bespoke" is not chosen.	

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denscale	Whether to scale the vertical range of the calendar age density plot relative to the calibration curve plot (optional). Default is 3 which means that the maximum calendar age density will be at 1/3 of the height of the plot.
plot_pretty	logical, defaulting to TRUE. If set TRUE then will select pretty plotting margins (that create sufficient space for axis titles and rotates y-axis labels). If FALSE will implement current user values.

## Value

A data frame with one column calendar\_age\_BP containing the calendar ages, and the other column probability containing the probability at that calendar age.

#### Examples

```
# Calibration of a single determination expressed as 14C age BP
calib <- CalibrateSingleDetermination(860, 35, intcal20)</pre>
plot(calib, type = "1", xlim = c(1000, 600))
# Incorporating an automated plot to visualise the calibration
CalibrateSingleDetermination(860, 35, intcal20, plot_output = TRUE)
# Calibration of a single (old) determination expressed as 14C age BP
calib <- CalibrateSingleDetermination(31020, 100, intcal20)</pre>
plot(calib, type = "1", xlim = c(36500, 34500))
# Calibration of a single (old) determination expressed as F14C concentration
calib <- CalibrateSingleDetermination(</pre>
    0.02103493, 0.0002618564, intcal20, F14C_inputs = TRUE)
plot(calib, type = "1", xlim = c(36500, 34500))
# Calibration of a single determination expressed as 14C age BP
# against SHCal20 (and creating an automated plot)
CalibrateSingleDetermination(1413, 25, shcal20, plot_output = TRUE)
# Implementing a bespoke confidence interval level and plot in AD
CalibrateSingleDetermination(
    1413,
    25,
    shcal20,
    plot_output = TRUE,
    plot_cal_age_scale = "AD",
    interval_width = "bespoke",
    bespoke_probability = 0.8)
# Changing denscale (so the calendar age density takes up less space)
CalibrateSingleDetermination(
    1413,
    25,
    shcal20,
    plot_output = TRUE,
    interval_width = "bespoke",
    bespoke_probability = 0.8,
```

```
denscale = 5)
```

cervus

## Example real-life data - Cervus in Yukon and Alaska

## Description

63 radiocarbon determinations collated by Dale Guthrie, R. related to Cervus (wapiti) in Yukon and Alaska. Samples are restricted to those between 25,000–6000 <sup>14</sup>C yrs BP.

#### **Reference:**

Dale Guthrie, R. New carbon dates link climatic change with human colonization and Pleistocene extinctions. *Nature* **441**, 207–209 (2006). https://doi.org/10.1038/nature04604

## Usage

cervus

#### Format

cervus:

A data frame with 63 rows and 7 columns:

lab\_code The sample code for the <sup>14</sup>C laboratory

site\_code The site/museum code

location The location of the sample

**c14\_age** The observed <sup>14</sup>C ages of the samples (in <sup>14</sup>C yr BP)

c14\_sig The uncertainty in the observed <sup>14</sup>C ages reported by the radiocarbon laboratory

**f14c** The observed  $F^{14}C$  concentrations

f14c\_sig The uncertainty in the observed F<sup>14</sup>C concentrations reported by the radiocarbon laboratory

## Source

https://doi.org/10.1038/nature04604

equus

## Description

84 radiocarbon determinations collated by Dale Guthrie, R. related to Equus (horse) in Yukon and Alaska. Samples are restricted to those between 25,000–6000  $^{14}$ C yrs BP.

#### **Reference:**

Dale Guthrie, R. New carbon dates link climatic change with human colonization and Pleistocene extinctions. *Nature* **441**, 207–209 (2006). https://doi.org/10.1038/nature04604

#### Usage

equus

#### Format

equus:

A data frame with 84 rows and 7 columns:

**lab\_code** The sample code for the <sup>14</sup>C laboratory

site\_code The site/museum code

location The location of the sample

c14\_age The observed <sup>14</sup>C ages of the samples (in <sup>14</sup>C yr BP)

c14\_sig The uncertainty in the observed <sup>14</sup>C ages reported by the radiocarbon laboratory

**f14c** The observed  $F^{14}C$  concentrations

f14c\_sig The uncertainty in the observed F<sup>14</sup>C concentrations reported by the radiocarbon laboratory

## Source

https://doi.org/10.1038/nature04604

FindPosteriorMeanRate Find Posterior Mean Rate of Sample Occurrence for Poisson Process Model

#### Description

Given output from the Poisson process fitting function PPcalibrate calculate the posterior mean rate of sample occurrence (i.e., the underlying Poisson process rate  $\lambda(t)$ ) together with specified probability intervals, on a given calendar age grid (provided in cal yr BP).

Note: If you want to calculate and plot the result, use PlotPosteriorMeanRate instead.

For more information read the vignette:

vignette("Poisson-process-modelling", package = "carbondate")

## Usage

```
FindPosteriorMeanRate(
   output_data,
   calendar_age_sequence,
   n_posterior_samples = 5000,
   interval_width = "2sigma",
   bespoke_probability = NA,
   n_burn = NA,
   n_end = NA
)
```

#### Arguments

output_data	The return value from the updating function PPcalibrate. Optionally, the output
	data can have an extra list item named label which is used to set the label on
	the plot legend.
calendar_age_se	equence

A vector containing the calendar age grid (in cal yr BP) on which to calculate the posterior mean rate.

n\_posterior\_samples

Number of samples it will draw, after having removed n\_burn, from the (thinned) MCMC realisations stored in output\_data to estimate the rate  $\lambda(t)$ . These samples may be repeats if the number of, post burn-in, realisations is less than n\_posterior\_samples. If not given, 5000 is used.

interval\_width The confidence intervals to show for both the calibration curve and the predictive density. Choose from one of "1sigma" (68.3%), "2sigma" (95.4%) and "bespoke". Default is "2sigma".

bespoke\_probability

The probability to use for the confidence interval if "bespoke" is chosen above. E.g., if 0.95 is chosen, then the 95% confidence interval is calculated. Ignored if "bespoke" is not chosen.

sumed to be the total number of iterations performed, i.e. n\_iter, if not given.

n\_burn The number of MCMC iterations that should be discarded as burn-in (i.e., considered to be occurring before the MCMC has converged). This relates to the number of iterations (n\_iter) when running the original update functions (not the thinned output\_data). Any MCMC iterations before this are not used in the calculations. If not given, the first half of the MCMC chain is discarded. Note: The maximum value that the function will allow is n\_iter - 100 \* n\_thin (where n\_iter and n\_thin are the arguments that were given to PP-calibrate) which would leave only 100 of the (thinned) values in output\_data. The last iteration in the original MCMC chain to use in the calculations. As-

#### Value

A list, each item containing a data frame of the calendar\_age\_BP, the rate\_mean and the confidence intervals for the rate - rate\_ci\_lower and rate\_ci\_upper.

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### See Also

PlotPosteriorMeanRate

## Examples

```
# NOTE: All these examples are shown with a small n_iter and n_posterior_samples
# to speed up execution.
# Try n_iter and n_posterior_samples as the function defaults.
pp_output <- PPcalibrate(
    pp_uniform_phase$c14_age,
    pp_uniform_phase$c14_sig,
    intcal20,
    n_iter = 1000,
    show_progress = FALSE)
# Default plot with 2 sigma interval
FindPosteriorMeanRate(pp_output, seq(450, 640, length=10), n_posterior_samples = 100)
```

FindPredictiveCalendarAgeDensity

Find Predictive Estimate of Shared Calendar Age Density from Bayesian Non-Parametric DPMM Output

## Description

Given output from one of the Bayesian non-parametric summarisation functions (either PolyaUrn-BivarDirichlet or WalkerBivarDirichlet) calculate the predictive (summarised/shared) calendar age density and probability intervals on a given calendar age grid (provided in cal yr BP).

Note: If you want to calculate and plot the result, use PlotPredictiveCalendarAgeDensity instead.

## Usage

```
FindPredictiveCalendarAgeDensity(
   output_data,
   calendar_age_sequence,
   n_posterior_samples = 5000,
   interval_width = "2sigma",
   bespoke_probability = NA,
   n_burn = NA,
   n_end = NA
)
```

#### Arguments

output_data	The return value from one of the Bayesian non-parametric DPMM functions,
	e.g. PolyaUrnBivarDirichlet or WalkerBivarDirichlet, or a list, each item con-
	taining one of these return values. Optionally, the output data can have an extra
	list item named label which is used to set the label on the plot legend.

#### calendar\_age\_sequence

A vector containing the calendar age grid (in cal yr BP) on which to calculate the predictive (summarised/shared) density.

n\_posterior\_samples

Number of samples it will draw, after having removed n\_burn, from the (thinned) realisations stored in the DPMM outputs to estimate the predictive calendar age density. These samples may be repeats if the number of, post burn-in, realisations is less than n\_posterior\_samples. If not given, 5000 is used.

interval\_width The confidence intervals to show for both the calibration curve and the predictive density. Choose from one of "1sigma" (68.3%), "2sigma" (95.4%) and "bespoke". Default is "2sigma".

bespoke\_probability

The probability to use for the confidence interval if "bespoke" is chosen above. E.g., if 0.95 is chosen, then the 95% confidence interval is calculated. Ignored if "bespoke" is not chosen.

- n\_burn The number of MCMC iterations that should be discarded as burn-in (i.e., considered to be occurring before the MCMC has converged). This relates to the number of iterations (n\_iter) when running the original update functions (not the thinned output\_data). Any MCMC iterations before this are not used in the calculations. If not given, the first half of the MCMC chain is discarded. Note: The maximum value that the function will allow is n\_iter 100 \* n\_thin (where n\_iter and n\_thin are the arguments given to PolyaUrnBivarDirichlet or WalkerBivarDirichlet) which would leave only 100 of the (thinned) values in output\_data.
- n\_end The last iteration in the original MCMC chain to use in the calculations. Assumed to be the total number of iterations performed, i.e. n\_iter, if not given.

## Value

A data frame of the calendar\_age\_BP, the density\_mean and the confidence intervals for the density\_density\_ci\_lower and density\_ci\_upper.

#### See Also

PlotPredictiveCalendarAgeDensity

## Examples

# NOTE: All these examples are shown with a small n\_iter and n\_posterior\_samples

- # to speed up execution.
- # Try n\_iter and n\_posterior\_samples as the function defaults.
- # First generate output data

```
polya_urn_output <- PolyaUrnBivarDirichlet(
    two_normals$c14_age,
    two_normals$c14_sig,
    intcal20,
    n_iter = 100,
    show_progress = FALSE)
# Find results for example output, 2-sigma confidence interval (default)
FindPredictiveCalendarAgeDensity(
    polya_urn_output, seq(3600, 4700, length=12), n_posterior_samples = 500)</pre>
```

FindSummedProbabilityDistribution Find the summed probability distribution (SPD) for a set of radiocarbon observations

#### Description

Takes a set of radiocarbon determinations and uncertainties, independently calibrates each one, and then averages the resultant calendar age estimates to give the SPD estimate.

**Important:** This function should not be used for inference as SPDs are not statistically rigorous. Instead use either of:

- the Bayesian non-parametric summarisation approaches PolyaUrnBivarDirichlet or Walker-BivarDirichlet;
- or the Poisson process rate approach PPcalibrate

The SPD function provided here is only intended for comparison. We provide a inbuilt plotting option to show the SPD alongside the determinations and the calibration curve.

#### Usage

```
FindSummedProbabilityDistribution(
    calendar_age_range_BP,
    rc_determinations,
    rc_sigmas,
    calibration_curve,
    F14C_inputs = FALSE,
    resolution = 1,
    plot_output = FALSE,
    plot_cal_age_scale = "BP",
    interval_width = "2sigma",
    bespoke_probability = NA,
    denscale = 3,
    plot_pretty = TRUE
)
```

## Arguments

- 6			
	calendar_age_range_BP		
		A vector of length 2 with the start and end calendar age BP to calculate the SPD over. These two values must be in order with start value less than end value, e.g., (600, 1700) cal yr BP. The code will extend this range (by 400 cal yrs each side) for conservativeness	
	<pre>rc_determinatio</pre>	ns	
		A vector of observed radiocarbon determinations. Can be provided either as ${}^{14}C$ ages (in ${}^{14}C$ yr BP) or as $F^{14}C$ concentrations.	
	rc_sigmas	A vector of the (1-sigma) measurement uncertainties for the radiocarbon deter- minations. Must be the same length as rc_determinations and given in the same units.	
	calibration_cur	ve	
		A dataframe which must contain one column calendar_age_BP, and also columns c14_age and c14_sig or f14c and f14c_sig (or both sets). This format matches the curves supplied with this package, e.g., intcal20, intcal13, which contain all 5 columns.	
	F14C_inputs	TRUE if the provided rc_determinations are $F^{14}C$ concentrations and FALSE if they are radiocarbon ages. Defaults to FALSE.	
	resolution	The distance between the calendar ages at which to calculate the calendar age probability. Default is 1.	
	plot_output	TRUE if you wish to plot the determinations, the calibration curve, and the SPD on the same plot. Defaults to FALSE	
	plot_cal_age_sc	ale	
		Only for usage when plot_output = TRUE. The calendar scale to use for the x-axis. Allowed values are "BP", "AD" and "BC". The default is "BP", corresponding to plotting in cal yr BP.	
	interval_width	Only for usage when plot_output = TRUE. The confidence intervals to show for the calibration curve. Choose from one of "1sigma" (68.3%), "2sigma" (95.4%) and "bespoke". Default is "2sigma".	
	bespoke_probabi	lity	
		The probability to use for the confidence interval if "bespoke" is chosen above. E.g. if 0.95 is chosen, then the 95% confidence interval is calculated. Ignored if "bespoke" is not chosen.	
	denscale	Whether to scale the vertical range of the calendar age density plot relative to the calibration curve plot (optional). Default is 3 which means that the maximum SPD will be at 1/3 of the height of the plot.	
	plot_pretty	logical, defaulting to TRUE. If set TRUE then will select pretty plotting margins (that create sufficient space for axis titles and rotates y-axis labels). If FALSE will implement current user values.	

## Value

A data frame with one column calendar\_age\_BP containing the calendar ages, and the other column probability containing the probability at that calendar age

## GenerateOxcalCode

#### See Also

PolyaUrnBivarDirichlet, WalkerBivarDirichlet for rigorous non-parametric Bayesian alternatives; and PPcalibrate for a rigorous variable-rate Poisson process alternative.

#### Examples

```
# An example using 14C age BP and the IntCal 20 curve
SPD <- FindSummedProbabilityDistribution(</pre>
  calendar_age_range_BP=c(400, 1700),
  rc_determinations=c(602, 805, 1554),
  rc_sigmas=c(35, 34, 45),
   calibration_curve=intcal20)
plot(SPD, type = "1",
  xlim = rev(range(SPD$calendar_age_BP)),
  xlab = "Calendar Age (cal yr BP)")
# Using the inbuilt plotting features
SPD <- FindSummedProbabilityDistribution(</pre>
  calendar_age_range_BP=c(400, 1700),
  rc_determinations=c(602, 805, 1554),
  rc_sigmas=c(35, 34, 45),
  calibration_curve=intcal20,
  plot_output = TRUE,
  interval_width = "bespoke",
  bespoke_probability = 0.8)
# An different example using F14C concentrations and the IntCal 13 curve
SPD <- FindSummedProbabilityDistribution(</pre>
  calendar_age_range_BP=c(400, 2100),
  rc_determinations=c(0.8, 0.85, 0.9),
  rc_sigmas=c(0.01, 0.015, 0.012),
  F14C_inputs=TRUE,
  calibration_curve=intcal13)
plot(SPD, type = "1",
  xlim = rev(range(SPD$calendar_age_BP)),
  xlab = "Calendar Age (cal yr BP)")
```

Generate0xcalCode	Outputs code suitable for running in OxCal from a series of radiocar-
	bon determinations

#### Description

Outputs code suitable for running in OxCal from a series of radiocarbon determinations that can be given as either  ${}^{14}C$  age or  $F^{14}C$ .

## Usage

```
GenerateOxcalCode(
  model_name,
  rc_determinations,
  rc_sigmas,
  rc_names = NULL,
  F14C_inputs = FALSE,
  outfile_path = NULL
)
```

## Arguments

<pre>model_name</pre>	The name given to the model in the OxCal code.
<pre>rc_determinati</pre>	ons
	A vector of observed radiocarbon determinations. Can be provided either as ${}^{14}C$ ages (in ${}^{14}C$ yr BP) or as $F^{14}C$ concentrations.
rc_sigmas	A vector of the (1-sigma) measurement uncertainties for the radiocarbon deter- minations. Must be the same length as rc_determinations and given in the same units.
rc_names	Optional. The name of each data point - if given it must be the same length of rc_determinations.
F14C_inputs	TRUE if the provided rc_determinations are $F^{14}C$ concentrations and FALSE if they are radiocarbon ages. Defaults to FALSE.
outfile_path	Optional. If given the OxCal code will be output to the file at the path given, otherwise it will be output to the terminal.

## Value

None

## Examples

```
# Generate names automatically and outputs to the screen for 14C ages
GenerateOxcalCode("My_data", c(1123, 1128, 1135), c(32, 24, 25))
```

# Provide name automatically and outputs to the screen for F14C concentrations GenerateOxcalCode( "My\_data", c(0.832, 0.850, 0.846), c(0.004, 0.003, 0.009),

```
c("P-1", "P-2", "P-3"),
F14C_inputs=TRUE)
```

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human

## Description

46 radiocarbon determinations collated by Dale Guthrie, R. related to archaeological sites (i.e., evidence of human existence) in Alaska. Samples are restricted to those between 25,000-6000 <sup>14</sup>C yrs BP.

#### **Reference:**

Dale Guthrie, R. New carbon dates link climatic change with human colonization and Pleistocene extinctions. *Nature* **441**, 207–209 (2006). https://doi.org/10.1038/nature04604

## Usage

human

## Format

human:

A data frame with 46 rows and 7 columns:

**lab\_code** The sample code for the <sup>14</sup>C laboratory

site\_code The site/museum code

location The location of the sample

**c14\_age** The observed <sup>14</sup>C ages of the samples (in <sup>14</sup>C yr BP)

c14\_sig The uncertainty in the observed <sup>14</sup>C ages reported by the radiocarbon laboratory

**f14c** The observed  $F^{14}C$  concentrations

**f14c\_sig** The uncertainty in the observed F<sup>14</sup>C concentrations reported by the radiocarbon laboratory

#### Source

https://doi.org/10.1038/nature04604

intcal04

IntCal04 calibration curve

#### Description

The IntCal04 Northern Hemisphere radiocarbon age calibration curve on a calendar grid spanning from 26,000–0 cal yr BP (Before Present, 0 cal yr BP corresponds to 1950 CE).

*Note:* This dataset provides <sup>14</sup>C ages and  $F^{14}C$  values on a calendar age grid. This is different from the <sup>14</sup>C ages and  $\Delta^{14}C$  values provided in oxcal .14c files.

#### **Reference:**

PJ Reimer, MGL Baillie, E Bard, A Bayliss, JW Beck, C Bertrand, PG Blackwell, CE Buck, G Burr, KB Cutler, PE Damon, RL Edwards, RG Fairbanks, M Friedrich, TP Guilderson, KA Hughen, B Kromer, FG McCormac, S Manning, C Bronk Ramsey, RW Reimer, S Remmele, JR Southon, M Stuiver, S Talamo, FW Taylor, J van der Plicht, and CE Weyhenmeyer. 2004. Int-cal04 Terrestrial Radiocarbon Age Calibration, 0–26 Cal Kyr BP. *Radiocarbon* **46**(3):1029-1058 https://doi.org/10.1017/S0033822200032999.

#### Usage

intcal04

## Format

intcal04:

A data frame with 3,301 rows and 5 columns providing the IntCal04 radiocarbon age calibration curve on a calendar grid spanning from 26,000–0 cal yr BP:

**calendar\_age** The calendar age (in cal yr BP)

c14\_age The <sup>14</sup>C age (in <sup>14</sup>C yr BP)

**c14 sig** The  $(1-\sigma)$  uncertainty in the <sup>14</sup>C age

**f14c** The  ${}^{14}$ C age expressed as  $F^{14}$ C concentration

**f14c\_sig** The  $(1-\sigma)$  uncertainty in the F<sup>14</sup>C concentration

#### Source

https://doi.org/10.1017/S0033822200032999

intcal09

IntCal09 calibration curve

## Description

The IntCal09 Northern Hemisphere radiocarbon age calibration curve on a calendar grid spanning from 50,000–0 cal yr BP (Before Present, 0 cal yr BP corresponds to 1950 CE).

*Note:* This dataset provides <sup>14</sup>C ages and  $F^{14}C$  values on a calendar age grid. This is different from the <sup>14</sup>C ages and  $\Delta^{14}C$  values provided in oxcal .14c files.

## 18

#### intcal13

## **Reference:**

PJ Reimer, MGL Baillie, E Bard, A Bayliss, JW Beck, PG Blackwell, C Bronk Ramsey, CE Buck, GS Burr, RL Edwards, M Friedrich, PM Grootes, TP Guilderson, I Hajdas, TJ Heaton, AG Hogg, KA Hughen, KF Kaiser, B Kromer, FG McCormac, SW Manning, RW Reimer, DA Richards, JR Southon, S Talamo, CSM Turney, J van der Plicht, CE Weyhenmeyer. 2009. IntCal09 and Marine09 Radiocarbon Age Calibration Curves, 0–50,000 Years cal BP *Radiocarbon* **51**(4):1111-1150 https://doi.org/10.1017/S0033822200034202.

#### Usage

intcal09

## Format

intcal09:

A data frame with 3,521 rows and 5 columns providing the IntCal09 radiocarbon age calibration curve on a calendar grid spanning from 50,000–0 cal yr BP:

**calendar\_age** The calendar age (in cal yr BP) **c14\_age** The <sup>14</sup>C age (in <sup>14</sup>C yr BP) **c14\_sig** The (1- $\sigma$ ) uncertainty in the <sup>14</sup>C age **f14c** The <sup>14</sup>C age expressed as F<sup>14</sup>C concentration **f14c sig** The (1- $\sigma$ ) uncertainty in the F<sup>14</sup>C concentration

#### Source

http://doi.org/10.1017/S0033822200034202

intcal13

IntCal13 calibration curve

## Description

The IntCal13 Northern Hemisphere radiocarbon age calibration curve on a calendar grid spanning from 50,000–0 cal yr BP (Before Present, 0 cal yr BP corresponds to 1950 CE).

*Note:* This dataset provides <sup>14</sup>C ages and  $F^{14}C$  values on a calendar age grid. This is different from the <sup>14</sup>C ages and  $\Delta^{14}C$  values provided in oxcal .14c files.

#### **Reference:**

Reimer PJ, Bard E, Bayliss A, Beck JW, Blackwell PG, Bronk Ramsey C, Buck CE, Cheng H, Edwards RL, Friedrich M, Grootes PM, Guilderson TP, Haflidason H, Hajdas I, Hatt? C, Heaton TJ, Hogg AG, Hughen KA, Kaiser KF, Kromer B, Manning SW, Niu M, Reimer RW, Richards DA, Scott EM, Southon JR, Turney CSM, van der Plicht J. 2013. IntCal13 and Marine13 radiocarbon age

calibration curves 0-50000 years calBP. Radiocarbon 55(4) https://doi.org/10.2458/azu\_js\_rc.55.16947.

#### Usage

intcal13

## Format

intcal13:

A data frame with 5,141 rows and 5 columns providing the IntCal13 radiocarbon age calibration curve on a calendar grid spanning from 50,000–0 cal yr BP:

**calendar\_age** The calendar age (in cal yr BP) **c14\_age** The <sup>14</sup>C age (in <sup>14</sup>C yr BP) **c14\_sig** The (1- $\sigma$ ) uncertainty in the <sup>14</sup>C age **f14c** The <sup>14</sup>C age expressed as F<sup>14</sup>C concentration **f14c\_sig** The (1- $\sigma$ ) uncertainty in the F<sup>14</sup>C concentration

#### Source

http://doi.org/10.2458/azu\_js\_rc.55.16947

intcal20

IntCal20 calibration curve

#### Description

The IntCal20 Northern Hemisphere radiocarbon age calibration curve on a calendar grid spanning from 55,000–0 cal yr BP (Before Present, 0 cal yr BP corresponds to 1950 CE).

*Note:* This dataset provides <sup>14</sup>C ages and  $F^{14}C$  values on a calendar age grid. This is different from the <sup>14</sup>C ages and  $\Delta^{14}C$  values provided in oxcal .14c files.

#### **Reference:**

Reimer PJ, Austin WEN, Bard E, Bayliss A, Blackwell PG, Bronk Ramsey C, Butzin M, Cheng H, Edwards RL, Friedrich M, Grootes PM, Guilderson TP, Hajdas I, Heaton TJ, Hogg AG, Hughen KA, Kromer B, Manning SW, Muscheler R, Palmer JG, Pearson C, van der Plicht J, Reimer RW, Richards DA, Scott EM, Southon JR, Turney CSM, Wacker L, Adolphi F, Büntgen U, Capano M, Fahrni S, Fogtmann-Schulz A, Friedrich R, Köhler P, Kudsk S, Miyake F, Olsen J, Reinig F, Sakamoto M, Sookdeo A, Talamo S. 2020. The IntCal20 Northern Hemisphere radiocarbon age calibration curve (0-55 cal kBP). *Radiocarbon* **62** https://doi.org/10.1017/RDC.2020.41.

#### Usage

intcal20

#### intcal98

## Format

intcal20: A data frame with 9,501 rows and 5 columns providing the IntCal20 radiocarbon age calibration curve on a calendar grid spanning from 55,000–0 cal yr BP: **calendar\_age** The calendar age (in cal yr BP) **c14\_age** The <sup>14</sup>C age (in <sup>14</sup>C yr BP) **c14\_sig** The (1- $\sigma$ ) uncertainty in the <sup>14</sup>C age **f14c** The <sup>14</sup>C age expressed as F<sup>14</sup>C concentration **f14c\_sig** The (1- $\sigma$ ) uncertainty in the F<sup>14</sup>C concentration

#### Source

http://doi.org/10.1017/RDC.2020.41

intcal98 IntCal98 calibration curve

#### Description

The IntCal98 Northern Hemisphere radiocarbon age calibration curve on a calendar grid spanning from 24,000–0 cal yr BP (Before Present, 0 cal yr BP corresponds to 1950 CE).

*Note:* This dataset provides <sup>14</sup>C ages and  $F^{14}C$  values on a calendar age grid. This is different from the <sup>14</sup>C ages and  $\Delta^{14}C$  values provided in oxcal .14c files.

#### **Reference:**

M. Stuiver, P. J. Reimer, E. Bard, J. W. Beck, G. S. Burr, K. A. Hughen, B. Kromer, F. G. McCormac, J. v. d. Plicht and M. Spurk. 1998. INTCAL98 Radiocarbon Age Calibration, 24,000–0 cal BP. *Radiocarbon* **40**(3):1041-1083 https://doi.org/10.1017/S0033822200019123.

#### Usage

intcal98

#### Format

intcal98:

A data frame with 1,538 rows and 5 columns providing the IntCal98 radiocarbon age calibration curve on a calendar grid spanning from 24,000–0 cal yr BP:

calendar\_age The calendar age (in cal yr BP)

**c14\_age** The <sup>14</sup>C age (in <sup>14</sup>C yr BP)

**c14\_sig** The  $(1-\sigma)$  uncertainty in the <sup>14</sup>C age

**f14c** The  ${}^{14}$ C age expressed as  $F^{14}$ C concentration

**f14c\_sig** The (1- $\sigma$ ) uncertainty in the F<sup>14</sup>C concentration

#### Source

https://doi.org/10.1017/S0033822200019123

InterpolateCalibrationCurve

Interpolate a calibration curve at a set of calendar ages

#### Description

Interpolate a calibration curve at a set of calendar ages

#### Usage

```
InterpolateCalibrationCurve(
    new_calendar_ages_BP,
    calibration_curve,
    F14C_outputs = NA
)
```

#### Arguments

```
new_calendar_ages_BP
```

A scalar or vector containing calendar ages (in cal yr BP) at which to interpolate the values (both the means and uncertainties) of the given calibration curve. If not provided (and NA is given), will use the range from the minimum calendar age to the maximum calendar age of the original calibration curve spaced by 1.

calibration\_curve

A dataframe which must contain one column calendar\_age\_BP, and also columns c14\_age and c14\_sig or f14c and f14c\_sig (or both sets). This format matches the curves supplied with this package, e.g., intcal20, intcal13, which contain all 5 columns.

F14C\_outputs TRUE if only  $F^{14}C$  concentrations are required, FALSE if only the radiocarbon ages (in  $^{14}C$  yrs BP) are required and NA if both are required for the new curve.

## Value

A new dataframe with entries for the interpolated c14\_age, and c14\_sig, f14c and f14c\_sig values at the calendar\_age\_BP values that were given in new\_calendar\_age\_BP.

## Examples

# Interpolate intcal20 at a single calendar age. Generates both 14C ages and F14C scales. InterpolateCalibrationCurve(51020, intcal20)

# Interpolate intcal20 at two calendar ages. Generates F14C estimates only. InterpolateCalibrationCurve(c(51017, 51021), intcal20, TRUE) # Interpolate intcal20 at two calendar ages. Generate 14C age estimates (cal yr BP) only. InterpolateCalibrationCurve(c(51017, 51021), intcal20, FALSE)

# Interpolate intcal20 at every integer calendar age within the range of dates # (for intcal20 this is 0 to 55000 cal yr BP), and create estimates for both radiocarbon scales. cal\_curve <- InterpolateCalibrationCurve(NA, intcal20)</pre>

kerr

Example real-life data - Irish Rath

#### Description

255 radiocarbon determinations collated by Kerr and McCormick related to the building and use of raths in Ireland in the early-medieval period.

#### **Reference:**

Kerr, T., McCormick, F., 2014. Statistics, sunspots and settlement: influences on sum of probability curves. *Journal of Archaeological Science* **41**, 493–501.

#### Usage

kerr

## Format

kerr:

A data frame with 255 rows and 4 columns:

**c14\_age** The observed <sup>14</sup>C ages of the samples (in <sup>14</sup>C yr BP)

c14\_sig The uncertainty in the observed <sup>14</sup>C ages reported by the radiocarbon laboratory

**f14c** The observed  $F^{14}C$  concentrations

f14c\_sig The uncertainty in the observed F<sup>14</sup>C concentrations reported by the radiocarbon laboratory

#### Source

http://doi.org/10.1016/j.jas.2013.09.002

mammuthus

#### Description

117 radiocarbon determinations collated by Dale Guthrie, R. related to Mammuthus (mammoth) in Yukon and Alaska. Samples are restricted to those between 25,000–6000 <sup>14</sup>C yrs BP.

## **Reference:**

Dale Guthrie, R. New carbon dates link climatic change with human colonization and Pleistocene extinctions. *Nature* **441**, 207–209 (2006). https://doi.org/10.1038/nature04604

#### Usage

mammuthus

## Format

mammuthus:

A data frame with 117 rows and 7 columns:

**lab\_code** The sample code for the <sup>14</sup>C laboratory

site\_code The site/museum code

location The location of the sample

**c14\_age** The observed <sup>14</sup>C ages of the samples (in <sup>14</sup>C yr BP)

c14\_sig The uncertainty in the observed <sup>14</sup>C ages reported by the radiocarbon laboratory

**f14c** The observed  $F^{14}C$  concentrations

f14c\_sig The uncertainty in the observed F<sup>14</sup>C concentrations reported by the radiocarbon laboratory

#### Source

https://doi.org/10.1038/nature04604

PlotCalendarAgeDensityIndividualSample Plot Posterior Calendar Age Estimate for an Individual Determination after Joint Calibration

#### Description

Once a joint calibration function (any of PolyaUrnBivarDirichlet, WalkerBivarDirichlet or PPcalibrate) has been run to calibrate a set of related radiocarbon determinations, this function plots the posterior calendar age estimate for a given single determination. Shown are a (direct) histogram of the posterior calendar ages generated by the MCMC chain and also a (smoothed) kernel density estimate obtained using a Gaussian kernel. The highest posterior density (HPD) interval is also shown for the interval width specified (default  $2\sigma$ ).

For more information read the vignettes:

vignette("Non-parametric-summed-density", package = "carbondate")
vignette("Poisson-process-modelling", package = "carbondate")

**Note:** The output of this function will provide different results from independent calibration of the determination. By jointly, and simultaneously, calibrating all the related <sup>14</sup>C determinations using the library functions we are able to share the available calendar information between the samples. This should result in improved individual calibration.

## Usage

```
PlotCalendarAgeDensityIndividualSample(
  ident,
  output_data,
  calibration_curve = NULL,
  plot_14C_age = TRUE,
  plot_cal_age_scale = "BP",
  hist_resolution = 5,
  density_resolution = 1,
  interval_width = "2sigma",
  bespoke_probability = NA,
  n_burn = NA,
  n_{end} = NA,
  show_hpd_ranges = FALSE,
  show_unmodelled_density = FALSE,
  plot_pretty = TRUE
)
```

#### Arguments

ident	The individual determination for which you want to plot the posterior density estimate of the calendar age.	
output_data	The return value either from one of the Bayesian non-parametric DPMM func- tions (PolyaUrnBivarDirichlet or WalkerBivarDirichlet); or from the Poisson process modelling function (PPcalibrate).	
calibration_curve		
	This is usually not required since the name of the calibration curve variable is saved in the output data. However, if the variable with this name is no longer in your environment then you should pass the calibration curve here. If provided, this should be a dataframe which should contain at least 2 columns entitled	

this should be a dataframe which should contain at least 3 columns entitled calendar\_age, c14\_age and c14\_sig. This format matches intcal20.

plot_14C_age	Whether to use the radiocarbon age ( $^{14}$ C yr BP) as the units of the y-axis in the plot. Defaults to TRUE. If FALSE uses $F^{14}$ C concentration instead.
plot_cal_age_sc	cale
	The calendar scale to use for the x-axis. Allowed values are "BP", "AD" and "BC". The default is "BP" corresponding to plotting in cal yr BP.
hist_resolutior	1
	The distance between histogram breaks when plotting the individual posterior calendar age density. Default is 5.
density_resolut	tion
	The distance between calendar ages for the returned smoothed calendar age probability. Default is 1.
interval_width	The confidence intervals to show for the calibration curve and for the high- est posterior density ranges. Choose from one of "1sigma" (68.3%), "2sigma" (95.4%) and "bespoke". Default is "2sigma".
bespoke_probabi	ility
	The probability to use for the confidence interval if "bespoke" is chosen above. E.g., if $0.95$ is chosen, then the 95% confidence interval is calculated. Ignored if "bespoke" is not chosen.
n_burn	The number of MCMC iterations that should be discarded as burn-in (i.e., considered to be occurring before the MCMC has converged). This relates to the number of iterations (n_iter) when running the original update functions (not the thinned output_data). Any MCMC iterations before this are not used in the calculations. If not given, the first half of the MCMC chain is discarded. Note: The maximum value that the function will allow is n_iter - $100 * n_thin$ (where n_iter and n_thin are the arguments given to PolyaUrnBivarDirichlet or WalkerBivarDirichlet) which would leave only 100 of the (thinned) values in output_data.
n_end	The iteration number of the last sample in output_data to use in the calculations. Assumed to be the total number of (thinned) realisations stored if not given.
show_hpd_ranges	
	Set to TRUE to also show the highest posterior density (HPD) range on the plot.
show_unmodelled	
	Set to TRUE to also show the unmodelled density (i.e., the result of independent calibration using CalibrateSingleDetermination) on the plot. Default is FALSE.
plot_pretty	logical, defaulting to TRUE. If set TRUE then will select pretty plotting margins (that create sufficient space for axis titles and rotates y-axis labels). If FALSE will implement current user values.

## Value

A data frame with one column calendar\_age\_BP containing the calendar ages, and the other column probability containing the (smoothed) kernel density estimate of the probability at that calendar age.

## See Also

CalibrateSingleDetermination for independent calibration of a sample against a calibration curve.

#### PlotConvergenceData

### Examples

```
# NOTE 1: These examples are shown with a small n_iter to speed up execution.
# When you run ensure n_iter gives convergence (try function default).
# NOTE 2: The examples only show application to PolvaUrnBivarDirichlet output.
# The function can also be used with WalkerBivarDirichlet and PPcalibrate output.
polya_urn_output <- PolyaUrnBivarDirichlet(</pre>
    two_normals$c14_age,
    two_normals$c14_sig,
    intcal20,
    n_{iter} = 500,
    n_{thin} = 2,
    show_progress = FALSE)
# Result for 15th determination
PlotCalendarAgeDensityIndividualSample(15, polya_urn_output)
# Now change to show 1 sigma interval for HPD range and calibration curve
# and plot in yr AD
PlotCalendarAgeDensityIndividualSample(
    15,
    polya_urn_output,
    plot_cal_age_scale = "AD",
    interval_width = "1sigma",
    show_hpd_ranges = TRUE,
    show_unmodelled_density = TRUE)
# Plot and then assign the returned probability
posterior_dens <- PlotCalendarAgeDensityIndividualSample(15, polya_urn_output)</pre>
# Use this to find the mean posterior calendar age
weighted.mean(posterior_dens$calendar_age_BP, posterior_dens$probability)
```

PlotConvergenceData Plot KL Divergence of Predictive Density to Assess Convergence of Bayesian Non-Parametric DPMM Sampler

## Description

This plots the Kullback-Leibler (KL) divergence between a fixed (initial/baseline) predictive density and the predictive density calculated from later individual realisations in the MCMC run of one of the Bayesian non-parametric summarisation approach. The divergence from the initial predictive density is plotted as a function of the realisation/iteration number.

This aims to identify when the divergence, from the initial estimate of the shared  $f(\theta)$  to the current estimate, has begun to stabilise. Hence, to (informally) assess when the MCMC chain has converged to equilibrium for the shared, underlying, predictive  $f(\theta)$ .

For more information read the vignette: vignette("determining-convergence", package = "carbondate") PlotConvergenceData(output\_data, n\_initial = NA)

## Arguments

output_data	The return value from one of the Bayesian non-parametric DPMM summarisa- tion functions, i.e., PolyaUrnBivarDirichlet or WalkerBivarDirichlet.
n_initial	The number of (thinned) realisations to use for the 'initial' predictive shared density. This predictive density is then compared with the predictive obtained at each subsequent realisation in the (thinned) DPMM output. If not specified, then the minimum of 1000 realisations, or 1 / 10 of the total number of realisations, will be used.

#### Value

None

## Examples

```
# Plot results for the example two_normal data
# NOTE: This does not show meaningful results as n_iter
# is too small. Try increasing n_iter to 1e5.
polya_urn_output <- PolyaUrnBivarDirichlet(
    two_normals$c14_age,
    two_normals$c14_sig,
    intcal20,
    n_iter = 500,
    show_progress = FALSE)
PlotConvergenceData(polya_urn_output)</pre>
```

PlotGelmanRubinDiagnosticMultiChain

*Plot Histogram of the Gelman-Rubin Convergence Diagnostic for Multiple Independent MCMC Chains* 

#### Description

This plots a histogram of the potential scale reduction factors (PSRF) for each of the individual posterior calendar age estimates for multiple independent MCMC chains. Achieved by comparing the *within-chain* variance with the *between-chains* variance after n\_burn iterations. The PSRF of each sample's posterior calendar age is calculated. If the chain have converged to the target posterior distribution, then PSRF should be close to 1 for all of the samples (a stringent condition is that all values are less than 1.1).

For more information read the vignette: vignette("determining-convergence", package = "carbondate")

#### Usage

PlotGelmanRubinDiagnosticMultiChain(output\_data\_list, n\_burn = NA)

## Arguments

output\_data\_list

A list, each item containing the return value from one of the updating functions e.g. PolyaUrnBivarDirichlet, WalkerBivarDirichlet or PPcalibrate. The minimum number of elements in the list is 2.

n\_burn The number of MCMC iterations that should be discarded for burn-in. This relates to the total number of iterations n\_iter when running the original update functions (not the thinned output\_data). Any MCMC iterations before this are not used in the calculations of the PSRF. If not given, the first half of the MCMC chain is discarded. Note: The maximum value that the function will allow is n\_iter - 100 \* n\_thin (where n\_iter and n\_thin are the arguments that were given to PPcalibrate) which would leave only 100 of the (thinned) values in output\_data.

## Value

None

## Examples

```
# Plot results for the example data - n_iter is too small for convergence
# Try increasing n_iter to see the values of the PSRF decrease
po <- list()
for (i in 1:3) {
    set.seed(i)
    po[[i]] <- PolyaUrnBivarDirichlet(
        two_normals$c14_age,
        two_normals$c14_sig,
        intcal20,
        n_iter=400,
        show_progress = FALSE)
}
PlotGelmanRubinDiagnosticMultiChain(po)
```

PlotGelmanRubinDiagnosticSingleChain Plot Histogram of the Gelman-Rubin Convergence Diagnostic for a Single MCMC Chain

## Description

This plots a histogram of the potential scale reduction factors (PSRF) for each of the individual posterior calendar age estimates within a single MCMC chain. Achieved by splitting the chain into segments after n\_burn and comparing the *within-chain* variance with the *between-chains* variance of the segments. The PSRF of each sample's posterior calendar age is calculated. If the chain have converged to the target posterior distribution, then PSRF should be close to 1 for all of the samples (a stringent condition is that all values are less than 1.1).

For more information read the vignette: vignette("determining-convergence", package = "carbondate")

#### Usage

PlotGelmanRubinDiagnosticSingleChain(output\_data, n\_burn = NA, n\_segments = 3)

#### Arguments

output_data	The return value from one of the updating functions, e.g., PolyaUrnBivarDirichlet, WalkerBivarDirichlet or PPcalibrate.
n_burn	The number of MCMC iterations that should be discarded for burn-in. This relates to the total number of iterations n_iter when running the original update functions (not the thinned output_data). Any MCMC iterations before this are not used in the calculations of the PSRF. If not given, the first half of the MCMC chain is discarded. Note: The maximum value that the function will allow is n_iter - $100 * n_thin$ (where n_iter and n_thin are the arguments that were given to PPcalibrate) which would leave only 100 of the (thinned) values in output_data.
n_segments	The number of segments to split the chain into. Default is 3, must be a number between 2 and 10.

#### Value

None

## Examples

```
# Plot results for the example data - n_iter is too small for convergence
# Try increasing n_iter to see the values of the PSRF decrease
polya_urn_output <- PolyaUrnBivarDirichlet(
    two_normals$c14_age,
    two_normals$c14_sig,
    intcal20,
    n_iter = 500,
    show_progress = FALSE)
PlotGelmanRubinDiagnosticSingleChain(polya_urn_output)
```

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PlotNumberOfClusters Plot Number of Calendar Age Clusters Estimated in Bayesian Non-Parametric DPMM Output

## Description

Given output from one of the Bayesian non-parametric summarisation functions (either PolyaUrn-BivarDirichlet or WalkerBivarDirichlet) plot the estimated number of calendar age clusters represented by the <sup>14</sup>C samples.

For more information read the vignette: vignette("Non-parametric-summed-density", package = "carbondate")

## Usage

PlotNumberOfClusters(output\_data, n\_burn = NA, n\_end = NA)

## Arguments

output_data	The return value from one of the Bayesian non-parametric DPMM functions, e.g. PolyaUrnBivarDirichlet or WalkerBivarDirichlet, or a list, each item con- taining one of these return values. Optionally, the output data can have an extra list item named label which is used to set the label on the plot legend.
n_burn	The number of MCMC iterations that should be discarded as burn-in (i.e., con- sidered to be occurring before the MCMC has converged). This relates to the number of iterations (n_iter) when running the original update functions (not the thinned output_data). Any MCMC iterations before this are not used in the calculations. If not given, the first half of the MCMC chain is dis- carded. Note: The maximum value that the function will allow is n_iter - 100 * n_thin (where n_iter and n_thin are the arguments given to PolyaU- rnBivarDirichlet or WalkerBivarDirichlet) which would leave only 100 of the (thinned) values in output_data.
n_end	The last iteration in the original MCMC chain to use in the calculations. As- sumed to be the total number of iterations performed, i.e. n_iter, if not given.

#### Value

None

#### See Also

PlotPredictiveCalendarAgeDensity and PlotCalendarAgeDensityIndividualSample for more plotting functions using DPMM output.

### Examples

```
# NOTE: these examples are shown with a small n_iter to speed up execution.
# When you run ensure n_iter gives convergence (try function default).
```

```
polya_urn_output <- PolyaUrnBivarDirichlet(
   two_normals$c14_age,
   two_normals$c14_sig,
   intcal20,
   n_iter = 500,
   n_thin = 2,
   show_progress = FALSE)</pre>
```

PlotNumberOfClusters(polya\_urn\_output)

```
PlotNumberOfInternalChanges
```

*Plot Number of Changepoints in Rate of Sample Occurrence for Poisson Process Model* 

## Description

Given output from the Poisson process fitting function PPcalibrate, plot the posterior distribution for the number of internal changepoints in the underlying rate of sample occurrence (i.e., in  $\lambda(t)$ ) over the period under study.

```
For more information read the vignette:
vignette("Poisson-process-modelling", package = "carbondate")
```

#### Usage

```
PlotNumberOfInternalChanges(output_data, n_burn = NA, n_end = NA)
```

#### Arguments

output_data	The return value from the updating function <b>PPcalibrate</b> . Optionally, the output data can have an extra list item named label which is used to set the label on the plot legend.
n_burn	The number of MCMC iterations that should be discarded as burn-in (i.e., considered to be occurring before the MCMC has converged). This relates to the number of iterations (n_iter) when running the original update functions (not the thinned output_data). Any MCMC iterations before this are not used in the calculations. If not given, the first half of the MCMC chain is discarded. Note: The maximum value that the function will allow is n_iter - 100 * n_thin (where n_iter and n_thin are the arguments that were given to PP-calibrate) which would leave only 100 of the (thinned) values in output_data.
n_end	The last iteration in the original MCMC chain to use in the calculations. As- sumed to be the total number of iterations performed, i.e. n_iter, if not given.

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## PlotPosteriorChangePoints

#### Value

None

## Examples

```
# NOTE: This example is shown with a small n_iter to speed up execution.
# Try n_iter and n_posterior_samples as the function defaults.
```

```
pp_output <- PPcalibrate(
    pp_uniform_phase$c14_age,
    pp_uniform_phase$c14_sig,
    intcal20,
    n_iter = 1000,
    show_progress = FALSE)
```

PlotNumberOfInternalChanges(pp\_output)

#### PlotPosteriorChangePoints

Plot Calendar Ages of Changes in Rate of Sample Occurrence for Poisson Process Model

## Description

Given output from the Poisson process fitting function PPcalibrate, plot the posterior density estimates for the calendar ages at which there are internal changepoints in the rate of sample occurrence  $\lambda(t)$ . These density estimates are calculated **conditional** upon the number of internal changepoints within the period under study (which is specified as an input to the function).

Having conditioned on the number of changes, n\_change, the code will extract all realisations from the the posterior of the MCMC sampler which have that number of internal changepoints in the estimate of  $\lambda(t)$ . It will then provide density estimates for the (ordered) calendar ages of those internal changepoints. These density estimates are obtained using a Gaussian kernel.

# Note: These graphs will become harder to interpret as the specified number of changepoints increases

```
For more information read the vignette:
vignette("Poisson-process-modelling", package = "carbondate")
```

#### Usage

```
PlotPosteriorChangePoints(
   output_data,
   n_changes = c(1, 2, 3),
   plot_cal_age_scale = "BP",
   n_burn = NA,
   n_end = NA,
   kernel_bandwidth = NA
)
```

## Arguments

output_data	The return value from the updating function PPcalibrate. Optionally, the output data can have an extra list item named label which is used to set the label on the plot legend.
n_changes	Number of internal changepoints to condition on, and plot for. A vector which can contain at most 4 elements, with values in the range 1 to 6. If not given, then $c(1, 2, 3)$ will be used.
plot_cal_age_sc	cale
	(Optional) The calendar scale to use for the x-axis. Allowed values are "BP", "AD" and "BC". The default is "BP" corresponding to plotting in cal yr BP.
n_burn	The number of MCMC iterations that should be discarded as burn-in (i.e., considered to be occurring before the MCMC has converged). This relates to the number of iterations $(n_iter)$ when running the original update functions (not the thinned output_data). Any MCMC iterations before this are not used in the calculations. If not given, the first half of the MCMC chain is discarded. Note: The maximum value that the function will allow is n_iter - 100 $*$ n_thin (where n_iter and n_thin are the arguments that were given to PP-calibrate) which would leave only 100 of the (thinned) values in output_data.
n_end	The last iteration in the original MCMC chain to use in the calculations. As- sumed to be the total number of iterations performed, i.e. n_iter, if not given.
kernel_bandwidth	
	(Optional) The bandwidth used for the (Gaussian) kernel smoothing of the cal- endar age densities. If not given, then 1/50th of the overall calendar age range will be used.

## Value

None

## Examples

# NOTE: This example is shown with a small <code>n\_iter</code> to speed up execution.

# Try n\_iter and n\_posterior\_samples as the function defaults.

```
pp_output <- PPcalibrate(
    pp_uniform_phase$c14_age,
    pp_uniform_phase$c14_sig,
    intcal20,
    n_iter = 1000,
    show_progress = FALSE)</pre>
```

# Plot the posterior change points for only 2 or 3 internal changes
PlotPosteriorChangePoints(pp\_output, n\_changes = c(2, 3))

PlotPosteriorHeights Plot Heights of Segments in Rate of Sample Occurrence for Poisson Process Model

#### Description

Given output from the Poisson process fitting function PPcalibrate, plot the posterior density estimates for the heights (i.e., values) of the piecewise-constant rate  $\lambda(t)$  used to model sample occurrence. These density estimates are calculated **conditional** upon the number of internal changepoints within the period under study (which is specified as an input to the function).

Having conditioned on the number of changes, n\_change, the code will extract all realisations from the the posterior of the MCMC sampler which have that number of internal changepoints in the estimate of  $\lambda(t)$ . It will then provide density estimates for the heights (i.e., the value) of the rate function between each of the determined (ordered) changepoints. These density estimates are obtained using a Gaussian kernel.

# Note: These graphs will become harder to interpret as the specified number of changepoints increases

For more information read the vignette:

vignette("Poisson-process-modelling", package = "carbondate")

## Usage

```
PlotPosteriorHeights(
   output_data,
   n_changes = c(1, 2, 3),
   n_burn = NA,
   n_end = NA,
   kernel_bandwidth = NA
)
```

#### Arguments

output_data	The return value from the updating function <b>PPcalibrate</b> . Optionally, the output data can have an extra list item named label which is used to set the label on the plot legend.
n_changes	Number of internal changepoints to condition on, and plot for. A vector which can contain at most 4 elements, with values in the range 1 to 6. If not given, then $c(1, 2, 3)$ will be used.
n_burn	The number of MCMC iterations that should be discarded as burn-in (i.e., considered to be occurring before the MCMC has converged). This relates to the number of iterations (n_iter) when running the original update functions (not the thinned output_data). Any MCMC iterations before this are not used in the calculations. If not given, the first half of the MCMC chain is discarded. Note: The maximum value that the function will allow is n_iter - 100 * n_thin (where n_iter and n_thin are the arguments that were given to PP-calibrate) which would leave only 100 of the (thinned) values in output_data.

n_end	The last iteration in the original MCMC chain to use in the calculations. As-
	sumed to be the total number of iterations performed, i.e. n_iter, if not given.
kernel_bandwidt	h
	(Optional) The bandwidth used for the (Gaussian) kernel smoothing of the cal- endar age densities. If not given, 1/50th of the maximum height will be used.

## Value

None

## Examples

# NOTE: This example is shown with a small n\_iter to speed up execution. # Try n\_iter and n\_posterior\_samples as the function defaults.

```
pp_output <- PPcalibrate(
    pp_uniform_phase$c14_age,
    pp_uniform_phase$c14_sig,
    intcal20,
    n_iter = 1000,
    show_progress = FALSE)
# Plot the posterior heights for only 2 or 3 internal changes</pre>
```

PlotPosteriorHeights(pp\_output, n\_changes = c(2, 3))

PlotPosteriorMeanRate Plot Posterior Mean Rate of Sample Occurrence for Poisson Process Model

## Description

Given output from the Poisson process fitting function PPcalibrate calculate and plot the posterior mean rate of sample occurrence (i.e., the underlying Poisson process rate  $\lambda(t)$ ) together with specified probability intervals, on a given calendar age grid (provided in cal yr BP).

Will show the original set of radiocarbon determinations (those you are modelling/summarising), the chosen calibration curve, and the estimated posterior rate of occurrence  $\lambda(t)$  on the same plot. Can also optionally show the posterior mean of each individual sample's calendar age estimate.

**Note:** If all you are interested in is the value of the posterior mean rate on a grid, without an accompanying plot, you can use FindPosteriorMeanRate instead.

```
For more information read the vignette:
vignette("Poisson-process-modelling", package = "carbondate")
```

#### Usage

```
PlotPosteriorMeanRate(
    output_data,
    n_posterior_samples = 5000,
```

```
calibration_curve = NULL,
plot_14C_age = TRUE,
plot_cal_age_scale = "BP",
show_individual_means = TRUE,
show_confidence_intervals = TRUE,
interval_width = "2sigma",
bespoke_probability = NA,
denscale = 3,
resolution = 1,
n_burn = NA,
n_end = NA,
plot_pretty = TRUE
)
```

## Arguments

output\_data The return value from the updating function PPcalibrate. Optionally, the output data can have an extra list item named label which is used to set the label on the plot legend. n\_posterior\_samples Number of samples it will draw, after having removed n\_burn, from the (thinned) MCMC realisations stored in output\_data to estimate the rate  $\lambda(t)$ . These samples may be repeats if the number of, post burn-in, realisations is less than n\_posterior\_samples. If not given, 5000 is used. calibration\_curve This is usually not required since the name of the calibration curve variable is saved in the output data. However, if the variable with this name is no longer in your environment then you should pass the calibration curve here. If provided, this should be a dataframe which should contain at least 3 columns entitled calendar\_age, c14\_age and c14\_sig. This format matches intcal20. plot\_14C\_age Whether to use the radiocarbon age (<sup>14</sup>C yr BP) as the units of the y-axis in the plot. Defaults to TRUE. If FALSE uses F<sup>14</sup>C concentration instead. plot\_cal\_age\_scale (Optional) The calendar scale to use for the x-axis. Allowed values are "BP", "AD" and "BC". The default is "BP" corresponding to plotting in cal yr BP. show\_individual\_means (Optional) Whether to calculate and show the mean posterior calendar age estimated for each individual <sup>14</sup>C sample on the plot as a rug on the x-axis. Default is TRUE. show\_confidence\_intervals Whether to show the pointwise confidence intervals (at chosen probability level) on the plot. Default is TRUE. interval\_width The confidence intervals to show for both the calibration curve and the predictive density. Choose from one of "1sigma" (68.3%), "2sigma" (95.4%) and "bespoke". Default is "2sigma". bespoke\_probability The probability to use for the confidence interval if "bespoke" is chosen above.

	E.g., if 0.95 is chosen, then the 95% confidence interval is calculated. Ignored if "bespoke" is not chosen.
denscale	(Optional) Whether to scale the vertical range of the Poisson process mean rate plot relative to the calibration curve plot. Default is 3 which means that the maximum of the mean rate will be at 1/3 of the height of the plot.
resolution	The distance between calendar ages at which to calculate the value of the rate $\lambda(t)$ . These ages will be created on a regular grid that automatically covers the calendar period specified in output_data. Default is 1.
n_burn	The number of MCMC iterations that should be discarded as burn-in (i.e., considered to be occurring before the MCMC has converged). This relates to the number of iterations (n_iter) when running the original update functions (not the thinned output_data). Any MCMC iterations before this are not used in the calculations. If not given, the first half of the MCMC chain is discarded. Note: The maximum value that the function will allow is n_iter - 100 $\times$ n_thin (where n_iter and n_thin are the arguments that were given to PP-calibrate) which would leave only 100 of the (thinned) values in output_data.
n_end	The last iteration in the original MCMC chain to use in the calculations. As- sumed to be the total number of iterations performed, i.e. n_iter, if not given.
plot_pretty	logical, defaulting to TRUE. If set TRUE then will select pretty plotting margins (that create sufficient space for axis titles and rotates y-axis labels). If FALSE will implement current user values.

## Value

A list, each item containing a data frame of the calendar\_age\_BP, the rate\_mean and the confidence intervals for the rate - rate\_ci\_lower and rate\_ci\_upper.

## Examples

```
# NOTE: All these examples are shown with a small n_iter and n_posterior_samples
# to speed up execution.
# Try n_iter and n_posterior_samples as the function defaults.
pp_output <- PPcalibrate(</pre>
   pp_uniform_phase$c14_age,
   pp_uniform_phase$c14_sig,
   intcal20,
   n_{iter} = 1000,
    show_progress = FALSE)
# Default plot with 2 sigma interval
PlotPosteriorMeanRate(pp_output, n_posterior_samples = 100)
# Specify an 80% confidence interval
PlotPosteriorMeanRate(
   pp_output,
    interval_width = "bespoke",
    bespoke_probability = 0.8,
    n_posterior_samples = 100)
```

PlotPredictiveCalendarAgeDensity

*Plot Predictive Estimate of Shared Calendar Age Density from Bayesian Non-Parametric DPMM Output* 

#### Description

Given output from one of the Bayesian non-parametric summarisation functions (either PolyaUrnBivarDirichlet or WalkerBivarDirichlet) calculate and plot the predictive (summarised/shared) calendar age density and probability intervals on a given calendar age grid (provided in cal yr BP).

Will show the original set of radiocarbon determinations (those you are summarising), the chosen calibration curve, and the summarised predictive calendar age density on the same plot. Can also optionally show the SPD estimate.

**Note:** If all you are interested in is the estimated value of the predictive density on a grid, without an accompanying plot, you can use FindPredictiveCalendarAgeDensity instead.

For more information read the vignette:

```
vignette("Non-parametric-summed-density", package = "carbondate")
```

#### Usage

```
PlotPredictiveCalendarAgeDensity(
  output_data,
  n_posterior_samples = 5000,
  calibration_curve = NULL,
  plot_14C_age = TRUE,
  plot_cal_age_scale = "BP",
  show_SPD = FALSE,
  show_confidence_intervals = TRUE,
  interval_width = "2sigma",
  bespoke_probability = NA,
  denscale = 3,
  resolution = 1,
  n_burn = NA,
  n_{end} = NA,
  plot_pretty = TRUE
)
```

#### Arguments

output_data	The return value from one of the Bayesian non-parametric DPMM functions,
	e.g. PolyaUrnBivarDirichlet or WalkerBivarDirichlet, or a list, each item con-
	taining one of these return values. Optionally, the output data can have an extra
	list item named label which is used to set the label on the plot legend.
n_posterior_s	amples

Number of samples it will draw, after having removed n\_burn, from the (thinned) realisations stored in the DPMM outputs to estimate the predictive calendar age

density. These samples may be repeats if the number of, post burn-in, realisations is less than n\_posterior\_samples. If not given, 5000 is used.

calibration\_curve

This is usually not required since the name of the calibration curve variable is saved in the output data. However, if the variable with this name is no longer in your environment then you should pass the calibration curve here. If provided, this should be a dataframe which should contain at least 3 columns entitled calendar\_age, c14\_age and c14\_sig. This format matches intcal20.

- plot\_14C\_age Whether to use the radiocarbon age ( $^{14}$ C yr BP) as the units of the y-axis in the plot. Defaults to TRUE. If FALSE uses  $F^{14}$ C concentration instead.
- plot\_cal\_age\_scale

The calendar scale to use for the x-axis. Allowed values are "BP", "AD" and "BC". The default is "BP" corresponding to plotting in cal yr BP.

show\_SPD Whether to calculate and show the summed probability distribution on the plot (optional). Default is FALSE.

show\_confidence\_intervals

Whether to show the pointwise confidence intervals (at the chosen probability level) on the plot. Default is TRUE.

interval\_width The confidence intervals to show for both the calibration curve and the predictive density. Choose from one of "1sigma" (68.3%), "2sigma" (95.4%) and "bespoke". Default is "2sigma".

bespoke\_probability

The probability to use for the confidence interval if "bespoke" is chosen above. E.g., if 0.95 is chosen, then the 95% confidence interval is calculated. Ignored if "bespoke" is not chosen.

- denscale Whether to scale the vertical range of the summarised calendar age density plot relative to the calibration curve plot (optional). Default is 3 which means that the maximum predictive density will be at 1/3 of the height of the plot.
- resolution The distance between calendar ages at which to calculate the predictive shared density. These ages will be created on a regular grid that automatically covers the calendar period of the given set of <sup>14</sup>C samples. Default is 1.
- n\_burn The number of MCMC iterations that should be discarded as burn-in (i.e., considered to be occurring before the MCMC has converged). This relates to the number of iterations (n\_iter) when running the original update functions (not the thinned output\_data). Any MCMC iterations before this are not used in the calculations. If not given, the first half of the MCMC chain is discarded. Note: The maximum value that the function will allow is n\_iter -100 \* n\_thin (where n\_iter and n\_thin are the arguments given to PolyaUrnBivarDirichlet or WalkerBivarDirichlet) which would leave only 100 of the (thinned) values in output\_data.
- n\_end The last iteration in the original MCMC chain to use in the calculations. Assumed to be the total number of iterations performed, i.e. n\_iter, if not given.
- plot\_pretty logical, defaulting to TRUE. If set TRUE then will select pretty plotting margins (that create sufficient space for axis titles and rotates y-axis labels). If FALSE will implement current user values.

## Value

A list, each item containing a data frame of the calendar\_age\_BP, the density\_mean and the confidence intervals for the density\_density\_ci\_lower and density\_ci\_upper for each set of output data.

## See Also

FindPredictiveCalendarAgeDensity if only interested in the estimated value of the predictive density on a grid; PlotNumberOfClusters and PlotCalendarAgeDensityIndividualSample for more plotting functions using DPMM output.

## Examples

```
# NOTE: All these examples are shown with a small n_iter and n_posterior_samples
# to speed up execution.
# Try n_iter and n_posterior_samples as the function defaults.
polya_urn_output <- PolyaUrnBivarDirichlet(</pre>
    two_normals$c14_age,
    two_normals$c14_sig,
    intcal20,
   n_{iter} = 500,
    show_progress = FALSE)
walker_output <- WalkerBivarDirichlet(</pre>
    two_normals$c14_age,
    two_normals$c14_sig,
    intcal20,
   n_iter = 500,
    show_progress = FALSE)
# Plot results for a single calibration
PlotPredictiveCalendarAgeDensity(polya_urn_output, n_posterior_samples = 50)
# Plot results from two calibrations on the same plot
PlotPredictiveCalendarAgeDensity(
    list(walker_output, polya_urn_output), n_posterior_samples = 50)
# Plot and show the 80% confidence interval, show the SPD, add a custom label
polya_urn_output$label = "My plot"
PlotPredictiveCalendarAgeDensity(
    polya_urn_output,
    n_posterior_samples = 50,
    interval_width = "bespoke",
    bespoke_probability = 0.8,
    show_SPD = TRUE)
```

PolyaUrnBivarDirichlet

Calibrate and Summarise Multiple Radiocarbon Samples via a Bayesian Non-Parametric DPMM (with Polya Urn Updating)

## Description

This function calibrates sets of multiple radiocarbon ( $^{14}$ C) determinations, and simultaneously summarises the resultant calendar age information. This is achieved using Bayesian non-parametric density estimation, providing a statistically-rigorous alternative to summed probability distributions (SPDs).

It takes as an input a set of <sup>14</sup>C determinations and associated  $1\sigma$  uncertainties, as well as the radiocarbon age calibration curve to be used. The samples are assumed to arise from an (unknown) shared calendar age distribution  $f(\theta)$  that we would like to estimate, alongside performing calibration of each sample.

The function models the underlying distribution  $f(\theta)$  as a Dirichlet process mixture model (DPMM), whereby the samples are considered to arise from an unknown number of distinct clusters. Fitting is achieved via MCMC.

It returns estimates for the calendar age of each individual radiocarbon sample; and broader output (including the means and variances of the underpinning calendar age clusters) that can be used by other library functions to provide a predictive estimate of the shared calendar age density  $f(\theta)$ .

For more information read the vignette: vignette("Non-parametric-summed-density", package = "carbondate")

**Note:** The library provides two slightly-different update schemes for the MCMC. In this particular function, updating of the DPMM is achieved by a Polya Urn approach (Neal 2000) This is our recommended updating approach based on testing. The alternative, slice-sampled, approach can be found at WalkerBivarDirichlet

## **References:**

Heaton, TJ. 2022. "Non-parametric Calibration of Multiple Related Radiocarbon Determinations and their Calendar Age Summarisation." *Journal of the Royal Statistical Society Series C: Applied Statistics* **71** (5):1918-56. https://doi.org/10.1111/rssc.12599.

Neal, RM. 2000. "Markov Chain Sampling Methods for Dirichlet Process Mixture Models." *Journal of Computational and Graphical Statistics* **9** (2):249 https://doi.org/10.2307/1390653.

## Usage

```
PolyaUrnBivarDirichlet(
  rc_determinations,
  rc_sigmas,
  calibration_curve,
  F14C_{inputs} = FALSE,
  n_{iter} = 1e+05,
  n_{thin} = 10,
  use_F14C_space = TRUE,
  slice_width = NA,
  slice_multiplier = 10,
  n_clust = min(10, length(rc_determinations)),
  show_progress = TRUE,
  sensible_initialisation = TRUE,
  lambda = NA,
  nu1 = NA,
  nu2 = NA,
```

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```
A = NA,
B = NA,
alpha_shape = NA,
alpha_rate = NA,
mu_phi = NA,
calendar_ages = NA
```

## Arguments

<pre>rc_determinatic</pre>	ons
	A vector of observed radiocarbon determinations. Can be provided either as ${}^{14}C$ ages (in ${}^{14}C$ yr BP) or as $F^{14}C$ concentrations.
rc_sigmas	A vector of the (1-sigma) measurement uncertainties for the radiocarbon deter- minations. Must be the same length as rc_determinations and given in the same units.
calibration_cur	rve
	A dataframe which must contain one column calendar_age_BP, and also columns c14_age and c14_sig or f14c and f14c_sig (or both sets). This format matches the curves supplied with this package, e.g., intcal20, intcal13, which contain all 5 columns.
F14C_inputs	TRUE if the provided <code>rc_determinations</code> are $F^{14}C$ concentrations and <code>FALSE</code> if they are radiocarbon ages. Defaults to <code>FALSE</code> .
n_iter	The number of MCMC iterations (optional). Default is 100,000.
n_thin	How much to thin the MCMC output (optional). Will store every n_thin <sup>th</sup> iteration. 1 is no thinning, while a larger number will result in more thinning. Default is 10. Must choose an integer greater than 1. Overall number of MCMC realisations stored will be $n_{out} = floor(n_{iter}/n_{thin}) + 1$ so do not choose n_thin too large to ensure there are enough samples from the posterior to use for later inference.
use_F14C_space	If TRUE (default) the calculations within the function are carried out in $F^{14}C$ space. If FALSE they are carried out in $^{14}C$ age space. We recommend selecting TRUE as, for very old samples, calibrating in $F^{14}C$ space removes the potential affect of asymmetry in the radiocarbon age uncertainty. <i>Note:</i> This flag can be set independently of the format/scale on which rc_determinations were originally provided.
slice_width	Parameter for slice sampling (optional). If not given a value is chosen intel- ligently based on the spread of the initial calendar ages. Must be given if sensible_initialisation is FALSE.
slice_multiplie	er
	Integer parameter for slice sampling (optional). Default is 10. Limits the slice size to slice_multiplier * slice_width.
n_clust	The number of clusters with which to initialise the sampler (optional). Must be less than the length of $rc_determinations$ . Default is 10 or the length of $rc_determinations$ if that is less than 10.
show_progress	Whether to show a progress bar in the console during execution. Default is TRUE.

#### sensible\_initialisation

Whether to use sensible values to initialise the sampler and an automated (adaptive) prior on  $\mu_{\phi}$  and (A, B) that is informed by the observed rc\_determinations. If this is TRUE (the recommended default), then all the remaining arguments below are ignored.

#### lambda, nu1, nu2

Hyperparameters for the prior on the mean  $\phi_j$  and precision  $\tau_j$  of each individual calendar age cluster j:

 $(\phi_i, \tau_i)|\mu_{\phi} \sim \text{NormalGamma}(\mu_{\phi}, \lambda, \nu_1, \nu_2)$ 

where  $\mu_{\phi}$  is the overall cluster centering. Required if sensible\_initialisation is FALSE.

A, B Prior on  $\mu_{\phi}$  giving the mean and precision of the overall centering  $\mu_{\phi} \sim N(A, B^{-1})$ . Required if sensible\_initialisation is FALSE.

alpha\_shape, alpha\_rate

Shape and rate hyperparameters that specify the prior for the Dirichlet Process (DP) concentration,  $\alpha$ . This concentration  $\alpha$  determines the number of clusters we expect to observe among our *n* sampled objects. The model places a prior on  $\alpha \sim \Gamma(\eta_1, \eta_2)$ , where  $\eta_1, \eta_2$  are the alpha\_shape and alpha\_rate. A small  $\alpha$  means the DPMM is more concentrated (i.e. we expect fewer calendar age clusters) while a large alpha means it is less less concentrated (i.e. many clusters). Required if sensible\_initialisation is FALSE.

- mu\_phi Initial value of the overall cluster centering  $\mu_{\phi}$ . Required if sensible\_initialisation is FALSE.
- calendar\_ages The initial estimate for the underlying calendar ages (optional). If supplied, it must be a vector with the same length as rc\_determinations. Required if sensible\_initialisation is FALSE.

#### Value

A list with 10 items. The first 8 items contain output of the model, each of which has one dimension of size  $n_{\text{out}} = \text{floor}(n_{\text{iter}}/n_{\text{thin}}) + 1$ . The rows in these items store the state of the MCMC from every  $n_{\text{thin}}^{\text{th}}$  iteration:

- cluster\_identifiers A list of length  $n_{out}$  each entry gives the cluster allocation (an integer between 1 and n\_clust) for each observation on the relevant MCMC iteration. Information on the state of these calendar age clusters (means and precisions) can be found in the other output items.
- alpha A double vector of length  $n_{out}$  giving the Dirichlet Process concentration parameter  $\alpha$ .
- n\_clust An integer vector of length  $n_{out}$  giving the current number of clusters in the model.
- phi A list of length  $n_{out}$  each entry giving a vector of length n\_clust of the means of the current calendar age clusters  $\phi_i$ .
- tau A list of length  $n_{out}$  each entry giving a vector of length n\_clust of the precisions of the current calenadar age cluster  $\tau_j$ .
- observations\_per\_cluster A list of length  $n_{out}$  each entry giving a vector of length n\_clust of the number of observations for that cluster.

- calendar\_ages An  $n_{out}$  by  $n_{obs}$  integer matrix. Gives the current estimate for the calendar age of each individual observation.
- mu\_phi A vector of length  $n_{out}$  giving the overall centering  $\mu_{\phi}$  of the calendar age clusters.

where  $n_{obs}$  is the number of radiocarbon observations i.e. the length of rc\_determinations.

The remaining items give information about the input data, input parameters (or those calculated using sensible\_initialisation) and the update\_type

update\_type A string that always has the value "Polya Urn".

input\_data A list containing the <sup>14</sup>C data used, and the name of the calibration curve used.

input\_parameters A list containing the values of the fixed hyperparameters lambda, nu1, nu2, A, B, alpha\_shape, alpha\_rate and mu\_phi, and the slice parameters slice\_width and slice\_multiplier.

## See Also

WalkerBivarDirichlet for our less-preferred MCMC method to update the Bayesian DPMM (otherwise an identical model); and PlotCalendarAgeDensityIndividualSample, PlotPredictiveCalendarAgeDensity and PlotNumberOfClusters to access the model output and estimate the calendar age information.

See also PPcalibrate for an an alternative (similarly rigorous) approach to calibration and summarisation of related radiocarbon determinations using a variable-rate Poisson process

#### Examples

```
# Note these examples are shown with a small n_iter to speed up execution.
# When you run ensure n_iter gives convergence (try function default).
# Basic usage making use of sensible initialisation to set most values and
# using a saved example data set and the IntCal20 curve.
polya_urn_output <- PolyaUrnBivarDirichlet(</pre>
    two_normals$c14_age,
    two_normals$c14_sig,
    intcal20,
    n_{iter} = 100,
    show_progress = FALSE)
# The radiocarbon determinations can be given as F14C concentrations
polya_urn_output <- PolyaUrnBivarDirichlet(</pre>
    two_normals$f14c,
    two_normals$f14c_sig,
    intcal20,
    F14C_inputs = TRUE,
    n_{iter} = 100,
    show_progress = FALSE)
```

PPcalibrate

Model Occurrence of Multiple Radiocarbon Samples as a Variable-Rate Poisson Process

## Description

This function calibrates a set of radiocarbon  $({}^{14}C)$  samples, and provides an estimate of how the underlying rate at which the samples occurred varies over calendar time (including any specific changepoints in the rate). The method can be used as an alternative approach to summarise calendar age information contained in a set of related  ${}^{14}C$  samples, enabling inference on the latent *activity* rate which led to their creation.

It takes as an input a set of <sup>14</sup>C determinations and associated  $1\sigma$  uncertainties, as well as the radiocarbon age calibration curve to be used. The (calendar age) occurrence of these radiocarbon (<sup>14</sup>C) samples is modelled as a Poisson process. The underlying rate of this Poisson process  $\lambda(t)$ , which represents the rate at which the samples occurred, is considered unknown and assumed to vary over calendar time.

Specifically, the sample occurrence rate  $\lambda(t)$  is modelled as piecewise constant, but with an unknown number of changepoints, which can occur at unknown times. The value of  $\lambda(t)$  between any set of changepoints is also unknown. The function jointly calibrates the given <sup>14</sup>C samples under this model, and simultaneously provides an estimate of  $\lambda(t)$ . Fitting is performed using reversible-jump MCMC within Gibbs.

It returns estimates for the calendar age of each individual radiocarbon sample in the input set; and broader output on the estimated value of  $\lambda(t)$  which can be used by other library functions. Analysis of the estimated changepoints in the piecewise  $\lambda(t)$  permits wider inference on whether the occurrence rate of samples changed significantly at any particular calendar time and, if so, when and by how much.

```
For more information read the vignette:
vignette("Poisson-process-modelling", package = "carbondate")
```

## Usage

```
PPcalibrate(
  rc_determinations,
  rc_sigmas,
  calibration_curve,
  F14C_inputs = FALSE,
  n_iter = 1e+05,
  n_thin = 10,
  use_F14C_space = TRUE,
  show_progress = TRUE,
  calendar_age_range = NA,
  calendar_grid_resolution = 1,
  prior_h_shape = NA,
  prior_h_rate = NA,
  prior_n_internal_changepoints_lambda = 3,
  k_max_internal_changepoints = 30,
```

#### PPcalibrate

```
rescale_factor_rev_jump = 0.9,
bounding_range_prob_cutoff = 0.001,
initial_n_internal_changepoints = 10,
grid_extension_factor = 0.1,
use_fast = TRUE,
fast_approx_prob_cutoff = 0.001
```

## Arguments

)

rc\_determinations A vector of observed radiocarbon determinations. Can be provided either as <sup>14</sup>C ages (in <sup>14</sup>C yr BP) or as F<sup>14</sup>C concentrations. A vector of the (1-sigma) measurement uncertainties for the radiocarbon deterrc\_sigmas minations. Must be the same length as rc\_determinations and given in the same units. calibration\_curve A dataframe which must contain one column calendar\_age\_BP, and also columns c14\_age and c14\_sig or f14c and f14c\_sig (or both sets). This format matches the curves supplied with this package, e.g., intcal20, intcal13, which contain all 5 columns. F14C\_inputs TRUE if the provided rc\_determinations are F<sup>14</sup>C concentrations and FALSE if they are radiocarbon ages. Defaults to FALSE. The number of MCMC iterations (optional). Default is 100,000. n\_iter n\_thin How much to thin the MCMC output (optional). Will store every n\_thin<sup>th</sup> iteration. 1 is no thinning, while a larger number will result in more thinning. Default is 10. Must choose an integer greater than 1. Overall number of MCMC realisations stored will be  $n_{out} = \text{floor}(n_{\text{iter}}/n_{\text{thin}}) + 1$  so do not choose n\_thin too large to ensure there are enough samples from the posterior to use for later inference. If TRUE (default) the calculations within the function are carried out in  $F^{14}C$ use\_F14C\_space space. If FALSE they are carried out in <sup>14</sup>C age space. We recommend selecting TRUE as, for very old samples, calibrating in F<sup>14</sup>C space removes the potential affect of asymmetry in the radiocarbon age uncertainty. Note: This flag can be set independently of the format/scale on which rc\_determinations were originally provided. show\_progress Whether to show a progress bar in the console during execution. Default is TRUE. calendar\_age\_range (Optional) Overall minimum and maximum calendar ages (in cal yr BP) permitted for the set of radiocarbon samples, i.e., calendar\_age\_range[1] <  $\theta_1, \ldots, \theta_n$ < calendar\_age\_range[1]. This is used to bound the start and end of the Poisson process (so no events will be permitted to occur outside this interval). If not selected then automated selection will be made based on given rc\_determinations and value of bounding\_range\_prob\_cutoff calendar\_grid\_resolution The spacing of the calendar age grid on which to restrict the potential calendar ages of the samples, e.g., calendar ages of samples are limited to being one of

<pre>t, t + resolution, t + 2 * resolution, Default is 1 (i.e., all calendar years in the overall calendar range are considered). Primarily used to speed-up code if have large range, when may select larger resolution. prior_h_shape, prior_h_rate (Optional) Prior for the value of the Poisson Process rate (the height rate_h) in any specific interval:</pre>
$rate\_h \sim Gamma(shape = prior\_h\_shape, rate = prior\_h\_rate).$
If they are both NA then prior_h_shape is selected to be 1 (so rate_h follows an Exponential distribution) and prior_h_rate chosen adaptively (internally) to match n_observations. prior_n_internal_changepoints_lambda Prior mean for the number of internal changepoints in the rate $\lambda(t)$ .
n_internal_changepoints $\sim Po(prior_n_internal_changepoints_lambda)$
k_max_internal_changepoints
Maximum permitted number of internal changepoints
rescale_factor_rev_jump
Factor weighting probability of dimension change in the reversible jump update step for Poisson process rate_h and rate_s
bounding_range_prob_cutoff
Probability cut-off for choosing the bounds for the potential calendar ages for the observations
initial_n_internal_changepoints
Number of internal changepoints to initialise MCMC sampler with. The de- fault is 10 (so initialise with diffuse state). Will place these initial changepoints uniformly at random within overall calendar age range.
grid_extension_factor
If you adaptively select the calendar_age_range from rc_determinations, how far you wish to extend the grid beyond this adaptive minimum and max- imum. The final range will be extended (equally on both sides) to cover (1 + grid_extension_factor) * (calendar_age_range)
use_fast, fast_approx_prob_cutoff
A flag to allow trimming the calendar age likelihood (i.e., reducing the range of calendar ages) for each individual sample to speed up the sampler. If TRUE (default), for each individual sample, those tail calendar ages (in the overall calendar_age_grid) with very small likelihoods will be trimmed (speeding up the updating of the calendar ages). If TRUE the probability cut-off to remove the tails is fast_approx_prob_cutoff.

## Value

A list with 7 items. The first 4 items contain output of the model, each of which has one dimension of size  $n_{\text{out}} = \text{floor}(n_{\text{iter}}/n_{\text{thin}}) + 1$ . The rows in these items store the state of the MCMC from every  $n_{\text{thin}}^{\text{th}}$  iteration:

rate\_s A list of length  $n_{out}$  each entry giving the current set of (calendar age) changepoint locations in the piecewise-constant rate  $\lambda(t)$ .

- rate\_h A list of length  $n_{out}$  each entry giving the current set of heights (values for the rate) in each piecewise-constant section of  $\lambda(t)$ .
- calendar\_ages An  $n_{out}$  by  $n_{obs}$  matrix. Gives the current estimate for the calendar age of each individual observation.
- n\_internal\_changes A vector of length  $n_{out}$  giving the current number of internal changes in the value of  $\lambda(t)$ .

where  $n_{obs}$  is the number of radiocarbon observations, i.e., the length of rc\_determinations.

The remaining items give information about input data, input parameters (or those calculated) and update\_type

update\_type A string that always has the value "RJPP".

input\_data A list containing the <sup>14</sup>C data used, and the name of the calibration curve used.

## Examples

# NOTE: This example is shown with a small n\_iter to speed up execution. # When you run ensure n\_iter gives convergence (try function default).

```
pp_output <- PPcalibrate(
    pp_uniform_phase$c14_age,
    pp_uniform_phase$c14_sig,
    intcal20,
    n_iter = 100,
    show_progress = FALSE)</pre>
```

pp\_uniform\_phase Example artificial data - Uniform Phase

#### Description

40 simulated radiocarbon determinations for which the underlying calendar ages are drawn (uniformly at random) from the period 550–500 cal yr BP.

$$f(\theta) = U[550, 500]$$

The observational uncertainty of each determination is set to be 15  $^{14}$ C yrs.

The corresponding  ${}^{14}C$  ages are then simulated based upon the IntCal20 calibration curve (convolved with the 15  ${}^{14}C$  yr measurement uncertainty):

$$X_i | \theta_i \sim N(m(\theta_i), \rho(\theta_i)^2 + 15^2),$$

where  $m(\theta_i)$  and  $\rho(\theta_i)$  are the IntCal20 pointwise means and uncertainties.

This dataset matches that used in the package vignette to illustrate the Poisson process modelling.

#### Usage

pp\_uniform\_phase

## Format

pp\_uniform\_phase:
A data frame with 40 rows and 4 columns:
c14\_age The simulated <sup>14</sup>C age (in <sup>14</sup>C yr BP)
c14\_sig The (fixed) <sup>14</sup>C age measurement uncertainty used in the simulation (set at 15 <sup>14</sup>C yrs)
f14c The corresponding simulated values of F<sup>14</sup>C concentration
f14c\_sig The (fixed) corresponding F<sup>14</sup>C measurement uncertainty used in the simulation

shcal04 SHCal04 calibration curve

## Description

The SHCal04 Southern Hemisphere radiocarbon age calibration curve on a calendar grid spanning from 11,000–0 cal yr BP (Before Present, 0 cal yr BP corresponds to 1950 CE).

*Note:* This dataset provides <sup>14</sup>C ages and  $F^{14}C$  values on a calendar age grid. This is different from the <sup>14</sup>C ages and  $\Delta^{14}C$  values provided in oxcal .14c files.

## **Reference:**

FG McCormac, AG Hogg, PG Blackwell, CE Buck, TFG Higham, and PJ Reimer 2004. SHCal04 Southern Hemisphere Calibration 0–11.0 cal kyr BP. *Radiocarbon* **46**(3):1087–1092 https://doi.org/10.1017/S003382220003.

#### Usage

shcal04

## Format

shcal04:

A data frame with 2,202 rows and 5 columns providing the SHCal04 radiocarbon age calibration curve on a calendar grid spanning from 11,000–0 cal yr BP: **calendar\_age** The calendar age (in cal yr BP)

**c14\_age** The  ${}^{14}$ C age (in  ${}^{14}$ C yr BP)

**c14\_sig** The  $(1-\sigma)$  uncertainty in the <sup>14</sup>C age

**f14c** The  ${}^{14}$ C age expressed as  $F^{14}$ C concentration

**f14c\_sig** The (1- $\sigma$ ) uncertainty in the F<sup>14</sup>C concentration

## Source

http://doi.org/10.1017/S0033822200033014

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shcal13

## Description

The SHCal13 Southern Hemisphere radiocarbon age calibration curve on a calendar grid spanning from 50,000–0 cal yr BP (Before Present, 0 cal yr BP corresponds to 1950 CE).

*Note:* This dataset provides <sup>14</sup>C ages and  $F^{14}C$  values on a calendar age grid. This is different from the <sup>14</sup>C ages and  $\Delta^{14}C$  values provided in oxcal .14c files.

## **Reference:**

Alan G Hogg, Quan Hua, Paul G Blackwell, Caitlin E Buck, Thomas P Guilderson, Timothy J Heaton, Mu Niu, Jonathan G Palmer, Paula J Reimer, Ron W Reimer, Christian S M Turney, Susan R H Zimmerman. 2013. SHCal13 Southern Hemisphere Calibration, 0-50,000 Years cal BP. *Radiocarbon* **55**(4):1889–1903 https://doi.org/10.2458/azu\_js\_rc.55.16783.

## Usage

shcal13

## Format

shcal13:

A data frame with 5,141 rows and 5 columns providing the SHCal13 radiocarbon age calibration curve on a calendar grid spanning from 50,000–0 cal yr BP:

**calendar\_age** The calendar age (in cal yr BP)

**c14\_age** The <sup>14</sup>C age (in <sup>14</sup>C yr BP)

**c14\_sig** The  $(1-\sigma)$  uncertainty in the <sup>14</sup>C age

**f14c** The <sup>14</sup>C age expressed as F<sup>14</sup>C concentration

**f14c\_sig** The (1- $\sigma$ ) uncertainty in the F<sup>14</sup>C concentration

## Source

http://doi.org/10.2458/azu\_js\_rc.55.16783

shcal20

## Description

The SHCal20 Southern Hemisphere radiocarbon age calibration curve on a calendar grid spanning from 55,000–0 cal yr BP (Before Present, 0 cal yr BP corresponds to 1950 CE).

*Note:* This dataset provides <sup>14</sup>C ages and  $F^{14}C$  values on a calendar age grid. This is different from the <sup>14</sup>C ages and  $\Delta^{14}C$  values provided in oxcal .14c files.

## **Reference:**

Hogg AG, Heaton TJ, Hua Q, Palmer JG, Turney CSM, Southon J, Bayliss A, Blackwell PG, Boswijk G, Bronk Ramsey C, Pearson C, Petchey F, Reimer P, Reimer R, Wacker L. 2020. SHCal20 Southern Hemisphere calibration, 0–55,000 years cal BP. *Radiocarbon* **62** https://doi.org/10.1017/RDC.2020.59.

#### Usage

shcal20

## Format

shcal20:

A data frame with 9,501 rows and 5 columns providing the SHCal20 radiocarbon age calibration curve on a calendar grid spanning from 55,000–0 cal yr BP:

calendar\_age The calendar age (in cal yr BP)

**c14\_age** The <sup>14</sup>C age (in <sup>14</sup>C yr BP)

**c14\_sig** The (1- $\sigma$ ) uncertainty in the <sup>14</sup>C age

**f14c** The <sup>14</sup>C age expressed as F<sup>14</sup>C concentration

**f14c\_sig** The (1- $\sigma$ ) uncertainty in the F<sup>14</sup>C concentration

#### Source

http://doi.org/10.1017/RDC.2020.59

two\_normals

## Description

50 simulated radiocarbon determinations for which the underlying calendar ages are drawn from a mixture of two normals:

$$f(\theta) = 0.45N(3500, 200^2) + 0.55N(5000, 100^2)$$

i.e., a mixture of a normal centred around 3500 cal yr BP; and another (slightly more concentrated/narrower) normal centred around 5000 cal yr BP.

The corresponding 50 radiocarbon ages were then simulated using the IntCal20 calibration curve incorporating both the uncertainty in the calibration curve and a hypothetical measurement uncertainty:

 $X_i | \theta_i \sim N(m(\theta_i), \rho(\theta_i)^2 + \sigma_{i, \text{lab}}^2),$ 

where  $m(\theta_i)$  and  $\rho(\theta_i)$  are the IntCal20 pointwise means and uncertainties; and  $\sigma_{i,\text{lab}}$ , the simulated laboratory measurement uncertainty, was fixed at a common value of 25 <sup>14</sup>C yrs.

This dataset is included simply to give some quick-to-run examples.

#### Usage

two\_normals

## Format

two\_normals:

A data frame with 50 rows and 4 columns:

**c14\_age** The simulated <sup>14</sup>C age (in <sup>14</sup>C yr BP)

c14\_sig The (fixed) <sup>14</sup>C age measurement uncertainty used in the simulation (set at 25 <sup>14</sup>C yrs)

f14c The corresponding simulated values of F<sup>14</sup>C concentration

f14c\_sig The (fixed) corresponding F<sup>14</sup>C measurement uncertainty used in the simulation

## Examples

```
# Plotting calendar age density underlying two_normals
# Useful for comparisons against estimation techniques
weights_true <- c(0.45, 0.55)
cluster_means_true_calBP <- c(3500, 5000)
cluster_precisions_true <- 1 / c(200, 100)^2
# Create mixture density
truedens <- function(t, w, truemean, trueprec) {
    dens <- 0
    for(i in 1:length(w)) {
```

```
dens <- dens + w[i] * dnorm(t, mean = truemean[i], sd = 1/sqrt(trueprec[i]))</pre>
 }
 dens
}
# Visualise mixture
curve(truedens(
 х,
 w = weights_true,
 truemean = cluster_means_true_calBP,
 trueprec = cluster_precisions_true),
 from = 2500, to = 7000, n = 401,
 xlim = c(7000, 2500),
 xlab = "Calendar Age (cal yr BP)",
 ylab = "Density",
 col = "red"
)
```

WalkerBivarDirichlet Calibrate and Summarise Multiple Radiocarbon Samples via a Bayesian Non-Parametric DPMM (with Walker Updating)

## Description

This function calibrates sets of multiple radiocarbon  $(^{14}C)$  determinations, and simultaneously summarises the resultant calendar age information. This is achieved using Bayesian non-parametric density estimation, providing a statistically-rigorous alternative to summed probability distributions (SPDs).

It takes as an input a set of <sup>14</sup>C determinations and associated  $1\sigma$  uncertainties, as well as the radiocarbon age calibration curve to be used. The samples are assumed to arise from an (unknown) shared calendar age distribution  $f(\theta)$  that we would like to estimate, alongside performing calibration of each sample.

The function models the underlying distribution  $f(\theta)$  as a Dirichlet process mixture model (DPMM), whereby the samples are considered to arise from an unknown number of distinct clusters. Fitting is achieved via MCMC.

It returns estimates for the calendar age of each individual radiocarbon sample; and broader output (the weights, means and variances of the underpinning calendar age clusters) that can be used by other library functions to provide a predictive estimate of the shared calendar age density  $f(\theta)$ .

For more information read the vignette: vignette("Non-parametric-summed-density", package = "carbondate")

**Note:** The library provides two slightly-different update schemes for the MCMC. In this particular function, updating of the DPMM is achieved by slice sampling (Walker 2007). We recommend use of the alternative to this, implemented at PolyaUrnBivarDirichlet

#### **Reference:**

Heaton, TJ. 2022. "Non-parametric Calibration of Multiple Related Radiocarbon Determinations

and their Calendar Age Summarisation." *Journal of the Royal Statistical Society Series C: Applied Statistics* **71** (5):1918-56. https://doi.org/10.1111/rssc.12599. Walker, SG. 2007. "Sampling the Dirichlet Mixture Model with Slices." *Communications in Statis*-

*tics - Simulation and Computation* **36** (1):45-54. https://doi.org/10.1080/03610910601096262.

## Usage

```
WalkerBivarDirichlet(
  rc_determinations,
  rc_sigmas,
  calibration_curve,
  F14C_inputs = FALSE,
  n_{iter} = 1e+05,
  n_{thin} = 10,
  use_F14C_space = TRUE,
  slice_width = NA,
  slice_multiplier = 10,
  show_progress = TRUE,
  sensible_initialisation = TRUE,
  lambda = NA,
  nu1 = NA,
  nu2 = NA,
  A = NA,
 B = NA,
  alpha_shape = NA,
  alpha_rate = NA,
 mu_phi = NA,
  calendar_ages = NA,
  n_clust = min(10, length(rc_determinations))
)
```

## Arguments

rc_determinations		
	A vector of observed radiocarbon determinations. Can be provided either as ${}^{14}C$ ages (in ${}^{14}C$ yr BP) or as $F^{14}C$ concentrations.	
rc_sigmas	A vector of the (1-sigma) measurement uncertainties for the radiocarbon deter- minations. Must be the same length as rc_determinations and given in the same units.	
calibration_curve		
	A dataframe which must contain one column calendar_age_BP, and also columns c14_age and c14_sig or f14c and f14c_sig (or both sets). This format matches the curves supplied with this package, e.g., intcal20, intcal13, which contain all 5 columns.	
F14C_inputs	TRUE if the provided rc_determinations are $F^{14}C$ concentrations and FALSE if they are radiocarbon ages. Defaults to FALSE.	
n_iter	The number of MCMC iterations (optional). Default is 100,000.	

n_thin	How much to thin the MCMC output (optional). Will store every n_thin <sup>th</sup> iteration. 1 is no thinning, while a larger number will result in more thinning. Default is 10. Must choose an integer greater than 1. Overall number of MCMC realisations stored will be $n_{out} = floor(n_{iter}/n_{thin}) + 1$ so do not choose n_thin too large to ensure there are enough samples from the posterior to use for later inference.
use_F14C_space	If TRUE (default) the calculations within the function are carried out in $F^{14}C$ space. If FALSE they are carried out in ${}^{14}C$ age space. We recommend selecting TRUE as, for very old samples, calibrating in $F^{14}C$ space removes the potential affect of asymmetry in the radiocarbon age uncertainty. <i>Note:</i> This flag can be set independently of the format/scale on which rc_determinations were originally provided.
slice_width	Parameter for slice sampling (optional). If not given a value is chosen intel- ligently based on the spread of the initial calendar ages. Must be given if sensible_initialisation is FALSE.
slice_multiplie	r
	Integer parameter for slice sampling (optional). Default is 10. Limits the slice size to slice_multiplier * slice_width.
<pre>show_progress sensible_initia</pre>	Whether to show a progress bar in the console during execution. Default is TRUE.
· · · · · <b>_</b> · · ·	Whether to use sensible values to initialise the sampler and an automated (adap-
	tive) prior on $\mu_{\phi}$ and (A, B) that is informed by the observed rc_determinations. If this is TRUE (the recommended default), then all the remaining arguments below are ignored.
lambda, nu1, nu2	
	Hyperparameters for the prior on the mean $\phi_j$ and precision $\tau_j$ of each individual calendar age cluster $j$ :
	$(\phi_j, \tau_j)   \mu_{\phi} \sim \text{NormalGamma}(\mu_{\phi}, \lambda, \nu_1, \nu_2)$
	where $\mu_{\phi}$ is the overall cluster centering. Required if sensible_initialisation is FALSE.
А, В	Prior on $\mu_{\phi}$ giving the mean and precision of the overall centering $\mu_{\phi} \sim N(A, B^{-1})$ . Required if sensible_initialisation is FALSE.
alpha_shape, al	pha_rate
	Shape and rate hyperparameters that specify the prior for the Dirichlet Process (DP) concentration, $\alpha$ . This concentration $\alpha$ determines the number of clusters we expect to observe among our $n$ sampled objects. The model places a prior on $\alpha \sim \Gamma(\eta_1, \eta_2)$ , where $\eta_1, \eta_2$ are the alpha_shape and alpha_rate. A small $\alpha$ means the DPMM is more concentrated (i.e. we expect fewer calendar age clusters) while a large alpha means it is less less concentrated (i.e. many clusters). Required if sensible_initialisation is FALSE.
mu_phi	Initial value of the overall cluster centering $\mu_{\phi}.$ Required if sensible_initialisation is FALSE.
calendar_ages	The initial estimate for the underlying calendar ages (optional). If supplied, it must be a vector with the same length as rc_determinations. Required if sensible_initialisation is FALSE.

n\_clust The number of clusters with which to initialise the sampler (optional). Must be less than the length of rc\_determinations. Default is 10 or the length of rc\_determinations if that is less than 10.

## Value

A list with 11 items. The first 8 items contain output of the model, each of which has one dimension of size  $n_{out} = \text{floor}(n_{\text{iter}}/n_{\text{thin}}) + 1$ . The rows in these items store the state of the MCMC from every  $n_{\text{thin}}^{\text{th}}$  iteration:

cluster\_identifiers An  $n_{out}$  by  $n_{obs}$  integer matrix. Provides the cluster allocation (an integer between 1 and n\_clust) for each observation on the relevant MCMC iteration. Information on the state of these calendar age clusters (means, precisions, and weights) can be found in the other output items.

alpha A double vector of length  $n_{out}$  giving the Dirichlet Process concentration parameter  $\alpha$ .

- n\_clust An integer vector of length  $n_{out}$  giving the current number of clusters in the model.
- phi A list of length  $n_{out}$  each entry giving a vector of the means of the current calendar age clusters  $\phi_j$ .
- tau A list of length  $n_{out}$  each entry giving a vector of the precisions of the current calendar age clusters  $\tau_i$ .
- weight A list of length  $n_{\text{out}}$  each entry giving the mixing weights of each calendar age cluster.
- calendar\_ages An  $n_{out}$  by  $n_{obs}$  integer matrix. Gives the current estimate for the calendar age of each individual observation.
- mu\_phi A vector of length  $n_{out}$  giving the overall centering  $\mu_{\phi}$  of the calendar age clusters.

where  $n_{obs}$  is the number of radiocarbon observations, i.e., the length of rc\_determinations.

The remaining items give information about the input data, input parameters (or those calculated using sensible\_initialisation) and the update\_type

update\_type A string that always has the value "Walker".

input\_data A list containing the  ${}^{14}C$  data used, and the name of the calibration curve used.

#### See Also

PolyaUrnBivarDirichlet for our preferred MCMC method to update the Bayesian DPMM (otherwise an identical model); and PlotCalendarAgeDensityIndividualSample, PlotPredictiveCalendarAgeDensity and PlotNumberOfClusters to access the model output and estimate the calendar age information.

See also PPcalibrate for an an alternative (similarly rigorous) approach to calibration and summarisation of related radiocarbon determinations using a variable-rate Poisson process

## Examples

```
# NOTE: These examples are shown with a small n_iter to speed up execution.
# When you run ensure n_iter gives convergence (try function default).
walker_output <- WalkerBivarDirichlet(</pre>
    two_normals$c14_age,
    two_normals$c14_sig,
    intcal20,
    n_{iter} = 100,
    show_progress = FALSE)
# The radiocarbon determinations can be given as F14C concentrations
walker_output <- WalkerBivarDirichlet(</pre>
    two_normals$f14c,
    two_normals$f14c_sig,
    intcal20,
    F14C_{inputs} = TRUE,
    n_{iter} = 100,
    show_progress = FALSE)
```

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