Employing **asremlPlus**, in conjunction with **asreml**, to calculate and use information criteria

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This vignette illustrates the facilities in asremlPlus (Brien, 2025), in conjunction with asreml (Butler et al., 2023), for calculating and using information. Here, asremlPlus and asreml are packages for the R Statistical Computing environment (R Core Team, 2025).

It is divided into the following main sections:

- 1. Set up the maximal model for this experiment
- 2. Obtaining information criteria for separate models
- 3. Obtaining information criteria for a prescribed sequence of model changes
- 4. Using information criteria to decide model changes

1. Set up the maximal model for this experiment

```
library(knitr)
opts_chunk$set("tidy" = FALSE, comment = NA)
suppressMessages(library(asreml, quietly=TRUE))
### Offline License checked out Thu Apr 3 13:19:21 2025
packageVersion("asreml")
### [1] '4.2.0.370'
suppressMessages(library(asremlPlus))
packageVersion("asremlPlus")
### [1] '4.4.47'
```

```
options(width = 100)
```

Get data available in asremlPlus

The data are from a 1976 spring wheat experiment and are taken from Gilmour et al. (1995). An analysis is presented in the asreml manual by Butler et al. (2023, Section 7.6), although they suggest that it is a barley experiment.

data(Wheat.dat)

Fit the maximal model

In the following a model is fitted that has the terms that would be included for a balanced lattice. In addition, a term WithinColPairs has been included to allow for extraneous variation arising between pairs of adjacent lanes. Also, separable ar1 residual autocorrelation has been included. This model represents the maximal anticipated model,

```
ASReml Version 4.2 03/04/2025 13:19:21
```

| | LogLik | Sigma2 | DF | wall | | |
|---|-----------|----------|-----|----------|---|---------------|
| 1 | -724.1213 | 23034.14 | 124 | 13:19:21 | | |
| 2 | -717.4149 | 9206.931 | 124 | 13:19:21 | (| 2 restrained) |
| 3 | -694.8752 | 26492.99 | 124 | 13:19:21 | (| 2 restrained) |
| 4 | -694.1600 | 33101.80 | 124 | 13:19:21 | (| 1 restrained) |
| 5 | -692.0020 | 36912.26 | 124 | 13:19:21 | (| 1 restrained) |
| 6 | -691.7892 | 46701.51 | 124 | 13:19:21 | (| 2 restrained) |
| 7 | -691.8336 | 46208.51 | 124 | 13:19:21 | (| 1 restrained) |
| 8 | -691.7749 | 47698.26 | 124 | 13:19:21 | | |
| 9 | -691.7711 | 47041.85 | 124 | 13:19:21 | | |
| | | | | | | |

Warning in asreml(yield ~ WithinColPairs + Variety, random = ~Rep/(Row + : Some components changed by more than 1% on the last iteration

The warning from asreml is probably due to a bound term.

Initialize a testing sequence by loading the current fit into an asrtests object

max.asrt <- as.asrtests(max.asr, NULL, NULL)</pre>

Check for and remove any boundary terms

max.asrt <- rmboundary(max.asrt)
summary(max.asrt\$asreml.obj)\$varcomp</pre>

| | component | std.error | z.ratio | bound %ch |
|-----------------------|--------------|--------------|------------|-----------|
| Rep:Row | 4.293282e+03 | 3.199458e+03 | 1.3418779 | P 0.0 |
| Rep:Column | 1.575689e+02 | 1.480357e+03 | 0.1064398 | P 0.7 |
| units | 5.742689e+03 | 1.652457e+03 | 3.4752438 | P 0.0 |
| Row:Column!R | 4.706787e+04 | 2.515832e+04 | 1.8708669 | P 0.0 |
| Row:Column!Row!cor | 7.920301e-01 | 1.014691e-01 | 7.8056280 | U 0.0 |
| Row:Column!Column!cor | 8.799559e-01 | 7.370402e-02 | 11.9390486 | U 0.0 |

```
print(max.asrt, which = "testsummary")
```

Sequence of model investigations for yield

(If a row has NA for p but not denDF, DF and denDF relate to fixed and variance parameter numbers)

terms DF denDF p AIC BIC action 1 Rep 1 NA NA NA NA Boundary

Rep has been removed because it has been constrained to zero. Following the recommendation of Littel et al. (2006, p. 150), the bound on all variance components is set to unconstrained (U) using setvariances.asreml so as to avoid bias in the estimate of the residual variance. Alternatively, one could move Rep to the fixed model.

Unbind Rep, Row and Column components and reload into an asrtests object

| <pre>max.asr <- setvarianceterms(max.asr\$call,</pre> | | | | | | | | |
|---|---------------|---------------|--------|----------|---|---|---------------|--|
| | | | | | | | | |
| ASRem | l Version 4.2 | 03/04/2025 13 | :19:23 | | | | | |
| | LogLik | Sigma2 | DF | wall | | | | |
| 1 | -724.1213 | 23034.14 | 124 | 13:19:23 | | | | |
| 2 | -717.4149 | 9206.931 | 124 | 13:19:23 | (| 2 | 2 restrained) | |
| 3 | -694.8752 | 26492.99 | 124 | 13:19:23 | (| 2 | 2 restrained) | |
| 4 | -693.9744 | 33129.65 | 124 | 13:19:23 | (| 1 | 1 restrained) | |
| 5 | -692.8856 | 39662.12 | 124 | 13:19:23 | | | | |
| 6 | -691.4276 | 53103.83 | 124 | 13:19:23 | | | | |
| 7 | -691.2387 | 48092.17 | 124 | 13:19:23 | | | | |
| 8 | -691.1808 | 47278.94 | 124 | 13:19:23 | | | | |
| 9 | -691.1710 | 46850.98 | 124 | 13:19:23 | | | | |
| 10 | -691.1700 | 46690.46 | 124 | 13:19:23 | | | | |
| Warning in asreml(fixed = yield ~ WithinColPairs + Variety, random = ~Rep/(Row + : Some comporchanged by more than 1% on the last iteration | | | | | | | | |

WARN [2025-04-03 13:19:23] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ WithinColPairs + Variety, random = -Rep/(Row + : Some components changed by more than 1% on the last iteration

WARN [2025-04-03 13:19:23] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ WithinColPairs + Variety, random = ~Rep/(Row + : Some components changed by more than 1% on the last iteration

max.asrt <- as.asrtests(max.asr, NULL, NULL)
max.asrt <- rmboundary(max.asrt)
summary(max.asrt\$asrem1.obj)\$varcomp</pre>

component std.error z.ratio bound %ch -2462.3785858 1.191435e+03 -2.066734 U 0.2 Rep Rep:Row 5012.4021415 3.396848e+03 1.475604 U 0.1 920.5936391 1.704008e+03 0.540252 U 1.1 Rep:Column units 5964.9099377 1.608792e+03 3.707695 P 0.1 Row:Column!R 46690.4620387 2.731906e+04 1.709080 P 0.0 Row:Column!Row!cor 0.8152180 9.988929e-02 8.161216 U 0.1 Row:Column!Column!cor 0.8857252 7.487875e-02 11.828793 U 0.0

print(max.asrt, which = "testsummary")

Sequence of model investigations for yield

(If a row has NA for p but not denDF, DF and denDF relate to fixed and variance parameter numbers)

[1] terms DF denDF p AIC BIC action
<0 rows> (or 0-length row.names)

Now the Rep component estimate is negative.

The test.summary output shows that no changes have been made to the model loaded using as.asrtests. The pseudo-anova table shows that Varieties are highly significant (p < 0.001)

2. Obtaining information criteria for separate models

The method infoCriteria has two methods for calculating information criteria. One, infoCriteria.asreml, is a method for asreml objects and the other, infoCriteria.list, if for 'listobjects, the components of the listbeing asreml' objects.

Single models

Firstly, infoCriteria is called with the default IClikelihood, which is REML. Then it is called with IClikelihood set to full (Verbyla, 2019).

infoCriteria(max.asr)

fixedDF varDF NBound AIC BIC loglik 1 0 7 0 1396.34 1416.082 -691.17

infoCriteria(max.asr, IClikelihood = "full")

ASReml Version 4.2 03/04/2025 13:19:24 LogLik Sigma2 DF wall 1 -691.1700 46641.98 124 13:19:24 Warning in asreml(fixed = yield ~ WithinColPairs + Variety, random = ~Rep/(Row + : Log-likelihood not converged

fixedDF varDF NBound AIC BIC loglik 1 26 7 0 1647.194 1746.545 -790.5968

A list of models

Now, a second model, from which the withinColPairs term has been omitted, is fitted; to be consistent, the variance components are unconstrained using setvariances.asreml. Then the asreml objects for this model and the maximal model are combined into a list and a data.frame produced that includes their information criteria.

```
ASReml Version 4.2 03/04/2025 13:19:24
```

| | LogLik | Sigma2 | DF | wall | | |
|---|-----------|----------|-----|----------|---|---------------|
| 1 | -727.7742 | 22898.99 | 125 | 13:19:24 | | |
| 2 | -721.0966 | 9190.303 | 125 | 13:19:24 | (| 2 restrained) |
| 3 | -698.3135 | 26671.76 | 125 | 13:19:24 | (| 2 restrained) |
| 4 | -697.5170 | 32677.28 | 125 | 13:19:24 | (| 1 restrained) |
| 5 | -695.4192 | 36662.27 | 125 | 13:19:24 | (| 1 restrained) |
| 6 | -695.2077 | 46263.96 | 125 | 13:19:24 | (| 2 restrained) |
| 7 | -695.1975 | 46156.63 | 125 | 13:19:24 | | |
| 8 | -695.1906 | 46630.21 | 125 | 13:19:24 | | |

Warning in asreml(yield ~ Variety, random = ~Rep/(Row + Column) + units, : Some components changed by more than 1% on the last iteration

ASReml Version 4.2 03/04/2025 13:19:24

| | LogLik | Sigma2 | DF | wall | | |
|----|-----------|----------|-----|----------|---|---------------|
| 1 | -727.7742 | 22898.99 | 125 | 13:19:24 | | |
| 2 | -721.0966 | 9190.303 | 125 | 13:19:24 | (| 2 restrained) |
| 3 | -698.3135 | 26671.76 | 125 | 13:19:24 | (| 2 restrained) |
| 4 | -697.3331 | 32689.33 | 125 | 13:19:24 | (| 1 restrained) |
| 5 | -697.0164 | 39975.97 | 125 | 13:19:24 | | |
| 6 | -695.0695 | 54825.30 | 125 | 13:19:24 | | |
| 7 | -694.7571 | 47637.20 | 125 | 13:19:24 | | |
| 8 | -694.6436 | 46775.41 | 125 | 13:19:24 | | |
| 9 | -694.6181 | 46175.06 | 125 | 13:19:24 | | |
| 10 | -694.6152 | 45940.69 | 125 | 13:19:24 | | |

Warning in asreml(fixed = yield ~ Variety, random = ~Rep/(Row + Column) + : Some components changed by more than 1% on the last iteration

WARN [2025-04-03 13:19:24] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = -Rep/(Row + Column) + : Some components changed by more than 1% on the last iteration

WARN [2025-04-03 13:19:25] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = -Rep/(Row + Column) + : Some components changed by more than 1% on the last iteration

```
mods <- list(max = max.asr, m1 = m1.asr)
ic <- infoCriteria(mods, IClikelihood = "full")
print(ic)</pre>
```

| | fixedDF | varDF | NBound | AIC | BIC | loglik |
|--------|---------|-------|--------|----------|----------|-----------|
| \max | 26 | 7 | 0 | 1647.194 | 1746.545 | -790.5968 |
| m1 | 25 | 7 | 0 | 1645.326 | 1741.666 | -790.6629 |

3. Obtaining information criteria for a prescribed sequence of model changes

The use of changeTerms.asrtests is demonstrated for a sequence of models, starting with the maximal model.

Drop the term for within Column pairs (a post hoc factor)

```
Warning in asreml(fixed = yield ~ Variety, random = ~Rep + units + Rep:Row + : Some components
changed by more than 1% on the last iteration
WARN [2025-04-03 13:19:26] Some components changed by more than 1% on the last iteration
Warning in asreml(fixed = yield ~ Variety, random = ~Rep + units + Rep:Row + : Some components
changed by more than 1% on the last iteration
print(current.asrt, which = "testsummary", omit.columns = "p")
```

Sequence of model investigations for yield

(If a row has NA for p but not denDF, DF and denDF relate to fixed and variance parameter numbers)

| | terms | DF | denDF | AIC | BIC | action |
|---|---------------------|----|-------|----------|----------|----------------|
| 1 | Maximal model | 26 | 7 | 1647.194 | 1746.545 | Starting model |
| 2 | Drop withinColPairs | 25 | 7 | 1645.326 | 1741.666 | Changed fixed |

So the same values of the information criteria have been obtained as when infoCriteria.list was used on a list containing the asreml objects for the two models. The differences is that here there is ultimately only one fitted model, the model stored in the asreml object in the asrtests object named current.asrt: this is the model with withinColPairs omitted.

Note this use of the omit.columns argument from print.test.summary to omit the irrelevant column p from the test.summary.

Drop nugget term

Check Row autocorrelation

4 Row autocorrelation 25

current.asrt <- changeTerms(current.asrt, newResidual = "Row:ar1(Column)",</pre> label="Row autocorrelation", IClikelihood = "full") WARN [2025-04-03 13:19:30] Some components changed by more than 1% on the last iteration Warning in asreml(fixed = yield ~ Variety, random = ~Rep + Rep:Row + Rep:Column, : Some components changed by more than 1% on the last iteration WARN [2025-04-03 13:19:30] Some components changed by more than 1% on the last iteration Warning in asreml(fixed = yield ~ Variety, random = ~Rep + Rep:Row + Rep:Column, : Some components changed by more than 1% on the last iteration WARN [2025-04-03 13:19:30] Log-likelihood not converged Warning in asreml(fixed = yield ~ Variety, random = ~Rep + Rep:Row + Rep:Column, : Log-likelihood not converged WARN [2025-04-03 13:19:30] Some components changed by more than 1% on the last iteration Warning in asreml(fixed = yield ~ Variety, random = ~Rep + Rep:Row + Rep:Column, : Some components changed by more than 1% on the last iteration WARN [2025-04-03 13:19:30] Log-likelihood not converged Warning in asreml(fixed = yield ~ Variety, random = ~Rep + Rep:Row + Rep:Column, : Log-likelihood not converged WARN [2025-04-03 13:19:30] Some components changed by more than 1% on the last iteration Warning in asreml(fixed = yield ~ Variety, random = ~Rep + Rep:Row + Rep:Column, : Some components changed by more than 1% on the last iteration Warning in newfit.asreml(asreml.obj, fixed. = fix.form, random. = ran.form, : print(current.asrt, which = "testsummary", omit.columns = "p") #### Sequence of model investigations for yield (If a row has NA for p but not denDF, DF and denDF relate to fixed and variance parameter numbers) terms DF denDF AIC BTC action Maximal model 26 7 1647.194 1746.545 Starting model 1 2 Drop withinColPairs 25 7 1645.326 1741.666 Changed fixed Drop units 25 6 1650.115 1743.445 Changed random 3

5 1660.788 1751.107 Changed residual - old uncoverged

4. Using information criteria to decide model changes

This sections illustrates the use of changeModelOnIC.asrtests to decide between consecutive models in a sequence of models. The default information criterion to use for this is the AIC. However, which.IC can be used to specify the use of the BIC or both. Here we use the AIC and the full likelihood.

Check the term for within Column pairs (a post hoc factor)

As before, we start with the maximal model, in which the variance components have been unconstrained and look to decide whether of not to drop the withinColPairs term.

Warning in asreml(fixed = yield ~ WithinColPairs + Variety, random = ~Rep/(Row + : Log-likelihood not converged

current.asrt <- iterate(current.asrt)</pre>

WARN [2025-04-03 13:19:32] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ WithinColPairs + Variety, random = ~Rep/(Row + : Some components changed by more than 1% on the last iteration

WARN [2025-04-03 13:19:32] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ WithinColPairs + Variety, random = ~Rep/(Row + : Some components changed by more than 1% on the last iteration

WARN [2025-04-03 13:19:33] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ WithinColPairs + Variety, random = ~Rep/(Row + : Some components changed by more than 1% on the last iteration

WARN [2025-04-03 13:19:33] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ WithinColPairs + Variety, random = -Rep/(Row + : Some components changed by more than 1% on the last iteration

WARN [2025-04-03 13:19:33] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep + units + Rep:Row + : Some components changed by more than 1% on the last iteration

WARN [2025-04-03 13:19:33] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep + units + Rep:Row + : Some components changed by more than 1% on the last iteration

print(current.asrt, which = "testsummary", omit.columns = "p")

Sequence of model investigations for yield

(If a row has NA for p but not denDF, DF and denDF relate to fixed and variance parameter numbers)

| | te | erms | DF | denDF | AIC | BIC | action |
|---|-------------|------|----|-------|-------------|-------------|----------------|
| 1 | Maximal mo | odel | 26 | 7 | 1647.193601 | 1746.544565 | Starting model |
| 2 | withinColPa | airs | -1 | 0 | -1.874126 | -4.884762 | Swapped |

Given the warning about a lack of convergence, we use iterate.asrtests to perform additional iterations of the fitting process. It seems that it was successful.

It can be seen from the **test.summary** that the term has been swapped out and this has the effect of reducing the number of fixed parameters by one and makes no change to the variance parameters.

Check the nugget term

WARN [2025-04-03 13:19:35] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep + units + Rep:Row + : Some components changed by more than 1% on the last iteration

WARN [2025-04-03 13:19:35] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep + units + Rep:Row + : Some components changed by more than 1% on the last iteration

WARN [2025-04-03 13:19:36] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep + Rep:Row + Rep:Column, : Some components changed by more than 1% on the last iteration

WARN [2025-04-03 13:19:36] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep + Rep:Row + Rep:Column, : Some components changed by more than 1% on the last iteration

Check Row autocorrelation

current.asrt <- changeModelOnIC(current.asrt, newResidual = "Row:ar1(Column)", label="Row autocorrelation", IClikelihood = "full", allow.unconverged = FALSE) WARN [2025-04-03 13:19:37] Some components changed by more than 1% on the last iteration Warning in asreml(fixed = yield ~ Variety, random = ~Rep + units + Rep:Row + : Some components changed by more than 1% on the last iteration WARN [2025-04-03 13:19:37] Some components changed by more than 1% on the last iteration Warning in asreml(fixed = yield ~ Variety, random = ~Rep + units + Rep:Row + : Some components changed by more than 1% on the last iteration WARN [2025-04-03 13:19:38] Log-likelihood not converged Warning in asreml(fixed = yield ~ Variety, random = ~Rep + units + Rep:Row + : Log-likelihood not converged WARN [2025-04-03 13:19:38] Some components changed by more than 1% on the last iteration Warning in asreml(fixed = yield ~ Variety, random = ~Rep + units + Rep:Row + : Some components changed by more than 1% on the last iteration WARN [2025-04-03 13:19:38] Log-likelihood not converged Warning in asreml(fixed = yield ~ Variety, random = ~Rep + units + Rep:Row + : Log-likelihood not converged WARN [2025-04-03 13:19:38] Some components changed by more than 1% on the last iteration Warning in asreml(fixed = yield ~ Variety, random = ~Rep + units + Rep:Row + : Some components changed by more than 1% on the last iteration Warning in newfit.asreml(asreml.obj, fixed. = fix.form, random. = ran.form, :

Check Column autocorrelation (depends on whether Row autocorrelation retained)

```
11
```

WARN [2025-04-03 13:19:39] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep + units + Rep:Row + : Some components changed by more than 1% on the last iteration

WARN [2025-04-03 13:19:40] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep + units + Rep:Row + : Some components changed by more than 1% on the last iteration

```
Warning in infoCriteria.asreml(asreml.obj, IClikelihood = ic.lik, bound.exclusions = bound.exclusions):
    Row:Column!Row!cor
```

Warning in infoCriteria.asreml(new.asrtests.obj\$asreml.obj, IClikelihood = ic.lik, : The following boun Row:Column!Row!cor

Output the results

print(current.asrt, which = "test", omit.columns = "p")

Sequence of model investigations for yield

(If a row has NA for p but not denDF, DF and denDF relate to fixed and variance parameter numbers)

| | terms | DF | denDF | AIC | BIC | action |
|-----|--------------------|----|-------|-------------|-------------|-----------------------------|
| 1 | Maximal model | 26 | 7 | 1647.193601 | 1746.544565 | Starting model |
| 2 | withinColPairs | -1 | 0 | -1.874126 | -4.884762 | Swapped |
| 3 | units | 0 | -1 | 4.789424 | 1.778789 | Unswapped |
| 4 R | ow autocorrelation | 0 | 0 | 0.00000 | 0.00000 | Unchanged - new unconverged |
| 5 C | ol autocorrelation | 0 | -2 | 19.478447 | 13.457177 | Unswapped |

summary(current.asrt\$asreml.obj)\$varcomp

| | component | std.error | z.ratio | bound | %ch |
|-----------------------|---------------|--------------|------------|-------|-----|
| Rep | -2392.1616314 | 1.199592e+03 | -1.9941460 | U | 0.4 |
| Rep:Row | 5033.2850607 | 3.408523e+03 | 1.4766764 | U | 0.2 |
| Rep:Column | 760.1498938 | 1.617038e+03 | 0.4700879 | U | 2.5 |
| units | 5929.0518909 | 1.609478e+03 | 3.6838361 | Р | 0.0 |
| Row:Column!R | 45940.6913910 | 2.634982e+04 | 1.7434920 | Р | 0.0 |
| Row:Column!Row!cor | 0.8101561 | 9.995026e-02 | 8.1055925 | U | 0.1 |
| Row:Column!Column!cor | 0.8846454 | 7.504265e-02 | 11.7885681 | U | 0.0 |

The test.summary shows us that the model without the autocorrelation failed to converge and so no change was made to the model. It, and the messages from checking the Column autocorrelation, also show us that the omission of the Column autocorrelation resulted in the Row autocorrelation becoming bound. That is, dropping the Column autocorrelation resulted in the dropping of two variance parameters

The function printFormulae.asreml is used to display the fitted model.

Formulae from asreml object

```
fixed: yield ~ Variety
random: ~ Rep + units + Rep:Row + Rep:Column
residual: ~ ar1(Row):ar1(Column)
```

References

Brien, C. J. (2025) asremlPlus: Augments ASReml-R in fitting mixed models and packages generally in exploring prediction differences. Version 4.4.47. https://cran.r-project.org/package=asremlPlus/ or http://chris.brien.name/rpackages/.

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Gilmour, A. R., Thompson, R., & Cullis, B. R. (1995). Average Information REML: An Efficient Algorithm for Variance Parameter Estimation in Linear Mixed Models. *Biometrics*, **51**, 1440–1450.

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Verbyla, A. P. (2019). A note on model selection using information criteria for general linear models estimated using REML. Australian & New Zealand Journal of Statistics, **61**, 39-50. https://doi.org/10.1111/anzs. 12254/.