Package ‘GaussianHMM1d’

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Title Inference, Goodness-of-Fit and Forecast for Univariate Gaussian Hidden Markov Models

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Description Inference, goodness-of-fit test, and prediction densities and intervals for univariate Gaussian Hidden Markov Models (HMM). The goodness-of-fit is based on a Cramer-von Mises statistic and uses parametric bootstrap to estimate the p-value. The description of the methodology is taken from Chapter 10.2 of Remillard (2013) <doi:10.1201/b14285>.

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R topics documented:

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em.step *Function to perform the E-M steps for the estimation of the paramters*

Description

This function perform the E-M steps for the estimation of the parameters of a univariate Gaussian HMM.

Usage

```
em.step(y, mu, sigma, Q)
```

Arguments

`y`	points at which the density function is comptuted (mx1);
`mu`	vector of means for each regime (r x 1);
`sigma`	vector of standard deviations for each regime (r x 1);
`Q`	transition probality matrix (r x r);

Value

`f`	values of the density function at time n+k
`w`	weights of the mixture

Author(s)

Bouchra R Nasri and Bruno N Rémillard, January 31, 2019

References

Chapter 10.2 of B. Rémillard (2013). Statistical Methods for Financial Engineering, Chapman and Hall/CRC Financial Mathematics Series, Taylor & Francis.

Examples

```
mu <- c(-0.3 ,0.7) ; sigma <- c(0.15,0.05); Q <- matrix(c(0.8, 0.3, 0.2, 0.7),2,2) ;
data <- Sim.HMM.Gaussian.1d(mu,sigma,Q,eta0=1,100)$x
out <- em.step(data,mu,sigma,Q)
```

EstHMM1d *Estimation of a univariate Gaussian Hidden Markov Model (HMM)*

Description

This function estimates parameters (mu, sigma, Q) of a univariate Hidden Markov Model. It computes also the probability of being in each regime, given the past observations (eta) and the whole series (lambda). The conditional distribution given past observations is applied to obtains pseudo-observations W that should be uniformly distributed under the null hypothesis. A Cramér-von Mises test statistic is then computed.

Usage

```
EstHMM1d(y, reg, max_iter = 10000, prec = 1e-04)
```

Arguments

`y`	(nx1) vector of data
`reg`	number of regimes
`max_iter`	maximum number of iterations of the EM algorithm; suggestion 10 000
`prec`	precision (stopping criteria); suggestion 0.0001.

Value

`mu`	estimated mean for each regime
`sigma`	stimated standard deviation for each regime
`Q`	(reg x reg) estimated transition matrix
`eta`	(n x reg) probabilities of being in regime k at time t given observations up to time t
`lambda`	(n x reg) probabilities of being in regime k at time t given all observations
`cvm`	Cramér-von Mises statistic for the goodness-of-fit test
`W`	Pseudo-observations that should be uniformly distributed under the null hypothesis of a Gaussian HMM
`LL`	Log-likelihood

Author(s)

Bouchra R Nasri and Bruno N Rémillard, January 31, 2019

References

Chapter 10.2 of B. Rémillard (2013). Statistical Methods for Financial Engineering, Chapman and Hall/CRC Financial Mathematics Series, Taylor & Francis.

Examples

```
Q <- matrix(c(0.8, 0.3, 0.2, 0.7),2,2); mu <- c(-0.3 ,0.7) ; sigma <- c(0.15,0.05)
data <- Sim.HMM.Gaussian.1d(mu,sigma,Q,eta0=1,100)$x
est <- EstHMM1d(data, 2, max_iter=10000, prec=0.0001)
```

EstRegime — *Estimated Regimes for the univariate Gaussian HMM*

Description

This function computes and plots the most likely regime for univariate Gaussian HMM using probabilities of being in regime k at time t given all observations (lambda) and probabilities of being in regime k at time t given observations up to time t (eta).

Usage

```
EstRegime(t, y, lambda, eta)
```

Arguments

`t`	(nx1) vector of dates (years, ...); if no dates then t=[1:length(y)]
`y`	(nx1) vector of data;
`lambda`	(nxreg) probabilities of being in regime k at time t given all observations;
`eta`	(nxreg) probabilities of being in regime k at time t given observations up to time t;

Value

`A`	Estimated Regime using lambda
`B`	Estimated Regime using eta
`runsA`	Estimated number of runs using lambda
`runsB`	Estimated number of runs using eta
`pA`	Graph for the estimated regime for each observation using lambda
`pB`	Graph for the estimated regime for each observation using eta

Author(s)

Bouchra R Nasri and Bruno N Rémillard, January 31, 2019

References

Chapter 10.2 of B. Rémillard (2013). Statistical Methods for Financial Engineering, Chapman and Hall/CRC Financial Mathematics Series, Taylor & Francis.

Examples

```
 Q <- matrix(c(0.8, 0.3, 0.2, 0.7),2,2); mu <- c(-0.3 ,0.7) ; sigma <- c(0.15,0.05);
data <- Sim.HMM.Gaussian.1d(mu,sigma,Q,eta0=1,100)$x
t=c(1:100);
est <- EstHMM1d(data, 2, max_iter=10000, prec=0.0001)
EstRegime(t,data,est$lambda, est$eta)
```

ForecastHMMeta — *Estimated probabilities of the regimes given new observations*

Description

This function computes the estimated probabilities of the regimes for a Gaussian HMM given new observation after time n. it also computes the associated weight of the Gaussian mixtures that can be used for forecasted density, cdf, or quantile function.

Usage

```
ForecastHMMeta(ynew, mu, sigma, Q, eta)
```

Arguments

ynew	new observations (mx1);
mu	vector of means for each regime (r x 1);
sigma	vector of standard deviations for each regime (r x 1);
Q	transition probality matrix (r x r);
eta	vector of the estimated probability of each regime (r x 1) at time n;

Value

etanew	values of the estimated probabilities at times n+1 to n+m, using the new observations
w	weights of the mixtures for periods n+1 to n+m

Author(s)

Bouchra R Nasri and Bruno N Rémillard, January 31, 2019

References

Chapter 10.2 of B. Rémillard (2013). Statistical Methods for Financial Engineering, Chapman and Hall/CRC Financial Mathematics Series, Taylor & Francis.

Examples

```
mu <- c(-0.3 ,0.7) ; sigma <- c(0.15,0.05); Q <- matrix(c(0.8, 0.3, 0.2, 0.7),2,2); eta <- c(.1,.9);
x <- c(0.2,-0.1,0.73)
out <- ForecastHMMeta(x,mu,sigma,Q,eta)
```

ForecastHMMPdf *Density function of a Gaussian HMM at time n+k*

Description

This function computes the density function of a Gaussian HMM at time n+k, given observation up to time n.

Usage

```
ForecastHMMPdf(x, mu, sigma, Q, eta, k)
```

Arguments

`x`	points at which the density function is comptuted (mx1);
`mu`	vector of means for each regime (r x 1);
`sigma`	vector of standard deviations for each regime (r x 1);
`Q`	transition probality matrix (r x r);
`eta`	vector of the estimated probability of each regime (r x 1) at time n;
`k`	time of prediction.

Value

`f`	values of the density function at time n+k
`w`	weights of the mixture

Author(s)

Bouchra R Nasri and Bruno N Rémillard, January 31, 2019

References

Chapter 10.2 of B. Rémillard (2013). Statistical Methods for Financial Engineering, Chapman and Hall/CRC Financial Mathematics Series, Taylor & Francis.

Examples

```
mu <- c(-0.3 ,0.7) ; sigma <- c(0.15,0.05); Q <- matrix(c(0.8, 0.3, 0.2, 0.7),2,2) ;
eta <- c(.9,.1);
x <- seq(-1, 1, by = 0.01)
out <- ForecastHMMPdf(x,mu,sigma,Q,eta,3)
plot(x,out$f,type="l")
```

GaussianMixtureCdf *Distribution function of a mixture of Gaussian univariate distributions*

Description

This function computes the distribution function of a mixture of Gaussian univariate distributions

Usage

```
GaussianMixtureCdf(x, mu, sigma, w)
```

Arguments

x	Points at which the distribution function is comptuted (nx1);
mu	vector of means for each regime (r x 1);
sigma	vector of standard deviations for each regime (r x 1);
w	vector of the probability of each regime (r x r).

Value

F	values of the distribution function

Author(s)

Bouchra R Nasri and Bruno N Rémillard, January 31, 2019

Examples

```
mu <- c(-0.3 ,0.7) ; sigma <- c(0.15,0.05); w <-c(0.8, 0.2);
x <- seq(-1, 1, by = 0.01)
F <- GaussianMixtureCdf(x,mu,sigma,w)
plot(x,F,type="l")
```

GaussianMixtureInv *Inverse distribution function of a mixture of Gaussian univariate distributions*

Description

This function computes the inverse distribution function of a mixture of Gaussian univariate distributions

Usage

```
GaussianMixtureInv(p, mu, sigma, w)
```

Arguments

p	Points in (0,1) at which the distribution function is comptuted (nx1);
mu	vector of means for each regime (r x 1);
sigma	vector of standard deviations for each regime (r x 1);
w	vector of the probability of each regime (r x 1).

Value

q	values of the quantile function

Author(s)

Bouchra R Nasri and Bruno N Rémillard, January 31, 2019

Examples

```
mu <- c(-0.3 ,0.7) ; sigma <- c(0.15,0.05); w <-c(0.8, 0.2);
p <- seq(0.01, 0.99, by = 0.01)
q <- GaussianMixtureInv(p,mu,sigma,w)
plot(p,q,type="l")
```

GaussianMixturePdf *Density function of a mixture of Gaussian univariate distributions*

Description

This function computes the density function of a mixture of Gaussian univariate distributions

Usage

```
GaussianMixturePdf(x, mu, sigma, w)
```

Arguments

x	Points at which the density is comptuted (n x 1);
mu	vector of means for each regime (r x 1);
sigma	vector of standard deviations for each regime (r x 1);
w	vector of the probability of each regime (r x 1).

Value

f	Values of the distribution function

Author(s)

Bouchra R Nasri and Bruno N Rémillard, January 31, 2019

Examples

```
mu <- c(-0.3 ,0.7) ; sigma <- c(0.15,0.05); w <-c(0.8, 0.2);
x <- seq(-1, 1, by = 0.01)
f <- GaussianMixturePdf(x,mu,sigma,w)
plot(x,f,type="l")
```

GofHMM1d *Goodness-of-fit test of a univariate Gaussian Hidden Markov Model*

Description

This function performs a goodness-of-fit test of a Gaussian HMM based on a Cramér-von Mises statistic using parametric bootstrap.

Usage

```
GofHMM1d(y, reg, max_iter = 10000, prec = 1e-04, n_sample = 1000,
  n_cores)
```

Arguments

y	(n x 1) data vector
reg	number of regimes
max_iter	maxmimum number of iterations of the EM algorithm; suggestion 10 000
prec	precision (stopping criteria); suggestion 0.0001
n_sample	number of bootstrap samples; suggestion 1000
n_cores	number of cores to use in the parallel computing

Value

pvalue	pvalue of the Cram\'er-von Mises statistic in percent
mu	estimated mean for each regime
sigma	estimated standard deviation for each regime
Q	(reg x reg) estimated transition matrix
eta	(n x reg) conditional probabilities of being in regime k at time t given observations up to time t
lambda	(n x reg) probabilities of being in regime k at time t given all observations
cvm	Cramér-von Mises statistic for the goodness-of-fit test
W	Pseudo-observations that should be uniformly distributed under the null hypothesis of a Gaussian HMM
LL	Log-likelihood

Author(s)

Bouchra R Nasri and Bruno N Rémillard, January 31, 2019

References

Chapter 10.2 of B. Rémillard (2013). Statistical Methods for Financial Engineering, Chapman and Hall/CRC Financial Mathematics Series, Taylor & Francis.

Examples

```
Q <- matrix(c(0.8, 0.3, 0.2, 0.7),2,2); mu <- c(-0.3 ,0.7) ; sigma <- c(0.15,0.05)
data <- Sim.HMM.Gaussian.1d(mu,sigma,Q,eta0=1,100)$x
gof <- GofHMM1d(data, 2, max_iter=10000, prec=0.0001, n_sample=100,n_cores=2)
```

Sim.HMM.Gaussian.1d   *Simulation of a univariate Gaussian Hidden Markov Model (HMM)*

Description

This function simulates observations from a univariate Gaussian HMM

Usage

```
Sim.HMM.Gaussian.1d(mu, sigma, Q, eta0, n)
```

Arguments

`mu`	vector of means for each regime (r x 1);
`sigma`	vector of standard deviations for each regime (r x 1);
`Q`	Transition probality matrix (r x r);
`eta0`	Initial value for the regime;
`n`	number of simulated observations.

Value

`x`	Simulated Data
`reg`	Markov chain regimes

Author(s)

Bouchra R Nasri and Bruno N Rémillard, January 31, 2019

Examples

```
Q <- matrix(c(0.8, 0.3, 0.2, 0.7),2,2) ; mu <- c(-0.3 ,0.7) ; sigma <- c(0.15,0.05);
sim <- Sim.HMM.Gaussian.1d(mu,sigma,Q,eta0=1,n=100)
```

`Sim.Markov.Chain` *Simulation of a finite Markov chain*

Description

This function generates a Markov chain X(1), ..., X(n) with transition matrix Q, starting from a state eta0.

Usage

```
Sim.Markov.Chain(Q, n, eta0)
```

Arguments

`Q`	Transition probality matrix (r x r);
`n`	length of series;
`eta0`	inital value in 1,...,r.

Value

`x`	Simulated Markov chain

Author(s)

Bouchra R Nasri and Bruno N Rémillard, January 31, 2019

Examples

```
Q <- matrix(c(0.8, 0.3, 0.2, 0.7),2,2) ;
 sim <- Sim.Markov.Chain(Q,eta0=1,n=100)
```

`Sn` *Cramer-von Mises statistic for goodness-of-fit of the null hypothesis of a univariate uniform distrubtion over [0,1]*

Description

This function computes the Cramér-von Mises statistic Sn for goodness-of-fit of the null hypothesis of a univariate uniform distrubtion over [0,1]

Usage

```
Sn(U)
```

Arguments

`U`	vector of pseudos-observations (apprimating uniform variates)

Value

`Sn`	Cramér-von Mises statistic

Author(s)

Bouchra R Nasri and Bruno N Rémillard, January 31, 2019

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