

# A simple graph system - **gRash**

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## 1 Introduction

This is a technical note which describes a simple “graph system” in **R** called **gRash**. The system is used in the **gRain** package for graphical independence networks. Thus **gRash** is not an **R** package but a part of an **R** package.

For the R community, the triplet triplet of the packages `graph`, `RBGL` and `Rgraphviz` constitutes tool for graph operations, manipulation and layout. The `gRash` system is not intended to be a strong competitor for these fine packages. On the contrary, part of the `gRash` functionality uses the other packages.

The main virtue of the `gRash` system is that graphs are specified in a way closer to normal text book representations and the same applies to some extent to the graph operations.

Only undirected and directed acyclic graphs are implemented.

## 2 Graphs

### 2.1 Undirected graphs

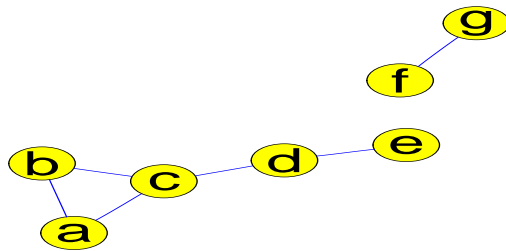
An undirected graph is created by the `newugsh` function. The graph can be specified by an incidence list in either of two different forms:

```
> ug1 <- newug(~a + b + c, ~c + d, ~d + e, ~f + g)
> ug1 <- newug(c("a", "b", "c"), c("c", "d"), c("d", "e"), c("f", "g"))
> ug1
```

```
Undirected graph
Nodes: a b c d e f g
Edges: a~b a~c b~c c~d d~e f~g
```

Graphs are displayed with `plot`:

```
> plot(ug1)
```



### 2.2 Directed acyclic graphs

A directed acyclic graph can be specified as:

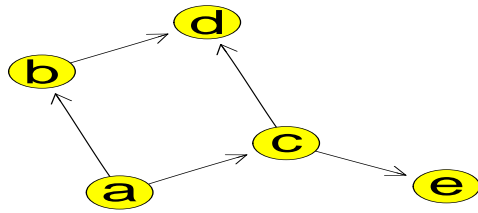
```
> dag1 <- newdag(~a, ~b + a, ~c + a, ~d + b + c, ~e + c)
> dag1 <- newdag("a", c("b", "a"), c("c", "a"), c("d", "b", "c"), c("e", "c"))
> dag1
```

```
Directed graph
Nodes: a b c d e
Edges: b<-a c<-a d<-b d<-c e<-c
```

Here `~a` means that “a” has no parents while `~d+b+c` means that “d” has parents “b” and “c”.

Graphs are displayed with `plot`:

```
> plot(dag1)
```



### 3 Operations on undirected graphs

#### 3.1 Graph queries

Many features of a graph are obtained by asking queries using the `queryg` function:

##### 3.1.1 Nodes

```
> queryg(ug1, "nodes")
a b c d e f g
```

##### 3.1.2 Edges

```
> queryg(ug1, "edges")
a b
a c
b c
c d
d e
f g
```

##### 3.1.3 Cliques

```
> queryg(ug1, "cliques")
c b a
c d
e d
f g
```

##### 3.1.4 Connected components

```
> queryg(ug1, "concomp")
a b c d e
f g
```

##### 3.1.5 Closure

```
> queryg(ug1, "cl", "c")
c a b d
```

### 3.1.6 Adjacencies

```
> queryg(ug1, "adj", "c")
```

```
a b d
```

### 3.1.7 Simplicial nodes

Nodes whose boundary is complete.

```
> queryg(ug1, "simplicialNodes")
```

```
a b e f g
```

### 3.1.8 Is complete

Is the graph complete?

```
> queryg(ug1, "is.complete")
```

```
[1] FALSE
```

### 3.1.9 Is simplicial

Is a node/set simplicial?

```
> queryg(ug1, "is.simplicial", "a")
```

```
[1] TRUE
```

```
> queryg(ug1, "is.simplicial", c("a", "b", "d"))
```

```
[1] FALSE
```

### 3.1.10 Is triangulated

```
> queryg(ug1, "is.triangulated")
```

```
[1] TRUE
```

### 3.1.11 Is $A$ and $B$ separated by $S$

```
> queryg(ug1, "separates", c("a", "b"), c("e", "f"), "d")
```

```
[1] TRUE
```

### 3.1.12 Subgraph

```
> queryg(ug1, "subgraph", c("a", "b", "c"))
```

```
Undirected graph  
Nodes: a b c  
Edges: c~b c~a b~a
```

## 3.2 Adjacency matrix

```
> convertg(ug1, to = "matrix")

      a      b      c      d      e      f      g
a FALSE TRUE  TRUE FALSE FALSE FALSE FALSE
b  TRUE FALSE  TRUE FALSE FALSE FALSE FALSE
c  TRUE  TRUE FALSE  TRUE FALSE FALSE FALSE
d FALSE FALSE  TRUE FALSE  TRUE FALSE FALSE
e FALSE FALSE FALSE  TRUE FALSE FALSE FALSE
f  FALSE FALSE FALSE FALSE FALSE FALSE  TRUE
g  FALSE FALSE FALSE FALSE FALSE  TRUE FALSE
```

## 3.3 Triangulation and Maximum Cardinality Search

### 3.3.1 Maximum cardinality search

Testing for whether a graph is triangulated is based on Maximum Cardinality Search:

```
> g <- newug(~a + b, ~b + c, ~c + d, ~d + e, ~e + a)
> mcs(g)
```

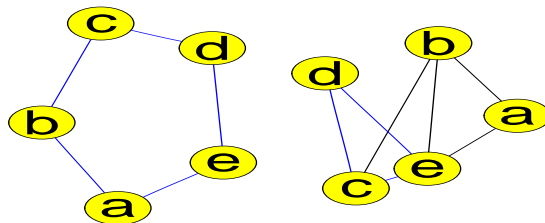
NULL

### 3.3.2 Triangulation

```
> tg <- triangulate(g)
> tg
```

Undirected graph  
Nodes: c d e b a  
Edges: c~d c~e d~e b~c b~e a~b a~e

```
> par(mfrow = c(1, 2))
> plot(g)
> plot(tg)
```



### 3.3.3 RIP ordering of the cliques

A RIP ordering of the cliques of a triangulated graph:

```
> rip <- ripOrder(tg)
> names(rip)
```

nodes cliques separators pa nLevels ch

```
> rip
```

```

Cliques
  1 c d e
  2 c e b
  3 e b a
Separators
  1 NA
  2 c e
  3 e b
Parents
  1 NA
  2 1
  3 2

```

## 4 Operations on directed acyclic graphs

### 4.1 Graph queries

Many features of a graph are obtained by asking queries using the `queryg` function as above:

#### 4.1.1 Parents

```

> queryg(dag1, "pa", "d")

b c

```

#### 4.1.2 Children

```

> queryg(dag1, "ch", "c")

d e

```

#### 4.1.3 Ancestral set

```

> queryg(dag1, "ancestralSet", c("b", "e"))

a b c e

```

### 4.2 Moralization

```

> moralize(dag1)

Undirected graph
Nodes: a b c d e
Edges: a~b a~c b~c b~d c~d c~e

```

### 4.3 Ancestral graph

```

> ancestralGraph(dag1, c("b", "e"))

Directed graph
Nodes: a b c e
Edges: e<-c c<-a b<-a

```

## 4.4 Checking for acyclicity

If a directed graph contains cycles, then NULL is returned

```
> newdag(~a + b, ~b + c, ~c + a)
```

```
NULL
```